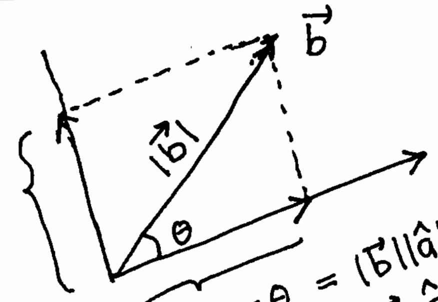
Decomposing a vector with respect to a direction (revisited)

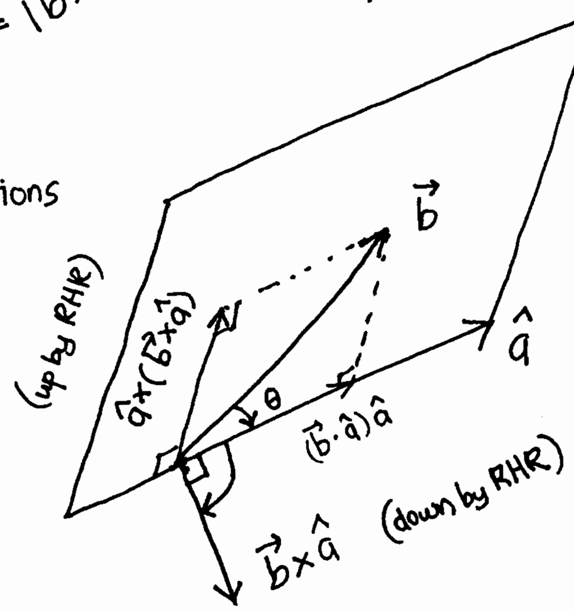
scalar projections

$$|\vec b|\sin\theta = |\vec b||\hat a|\sin\theta = |\vec b\times\hat a|$$



$$|\vec b|\cos\theta = |\vec b||\hat a|\cos\theta = \vec b\cdot\hat a$$

vector projections



$$\hat a\times(\vec b\times\hat a) = -\hat a\times(\hat a\times\vec b)$$

length:

$$|\hat a\times(\vec b\times\hat a)| = |\hat a||\vec b\times\hat a|\sin\tfrac{\pi}{2} = |\vec b\times\hat a|$$

direction:

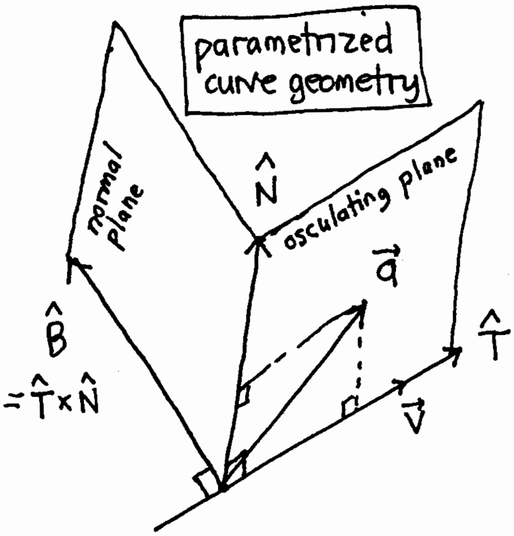
$$\widehat{\hat a\times(\vec b\times\hat a)} = \frac{\hat a\times(\vec b\times\hat a)}{|\vec b\times\hat a|} = \frac{\hat a\times(\vec b\times\vec a)}{|\vec b\times\vec a|}$$

vector identity: $\vec a\times(\vec b\times\vec c) = (\vec a\cdot\vec c)\,\vec b - (\vec a\cdot\vec b)\,\vec c$

special case: $\hat a\times(\vec b\times\hat a) = (\hat a\cdot\hat a)\,\vec b - (\hat a\cdot\vec b)\,\hat a$

$$= \vec b - \underbrace{(\vec b\cdot\hat a)\hat a}_{\overrightarrow{\mathrm{proj}}_{\hat a}\vec b} = \boxed{\overrightarrow{\mathrm{orthog}}_{\hat a}\vec b = -\hat a\times(\hat a\times\vec b)}$$

$\boxed{\text{parametrized curve geometry}}$



$$\hat B = \frac{\vec v\times\vec a}{|\vec v\times\vec a|} = \frac{\vec r\,'\times\vec r\,''}{|\vec r\,'\times\vec r\,''|} \quad \left(= \hat T\times\hat N\right)$$

$$\hat N = \hat B\times\hat T$$

compute $\hat B$ (easy)
take cross product to get $\hat N$