osculating circle in practice : parametrizing the space curve.

we don't have to do this in our level course, but it is important to realize this is not difficult to do.

$\boxed{\text{helix example}}$ we already evaluated:

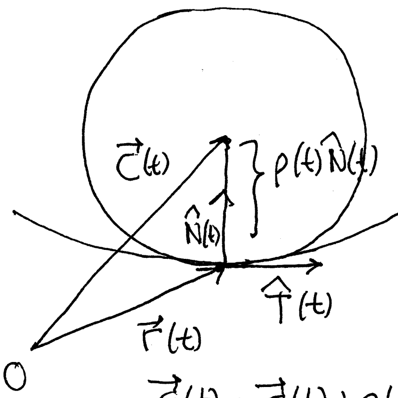
$\vec{r}(t) = \langle a\cos t, a\sin t, bt\rangle$

$\hat{T}(t) = \frac{1}{\sqrt{a^2+b^2}}\langle -a\sin t, a\cos t, b\rangle$

$\hat{N}(t) = \langle -\cos t, -\sin t, 0\rangle$

$\rho(t) = \frac{a^2+b^2}{a} > a$ "slinky stretch"

center of osculating circle:



$\vec{C}(t) = \vec{r}(t) + \rho(t)\,\hat{N}(t)$

$\vec{C}(t) = \langle a\cos t, a\sin t, bt\rangle$

$\quad + \frac{a^2+b^2}{a}\langle -\cos t, -\sin t, 0\rangle$

$\vec{R}(t,\theta) = \vec{C}(t) + \rho(t)\left[\cos\theta\,\hat{T}(t) + \sin\theta\,\hat{N}(t)\right]$

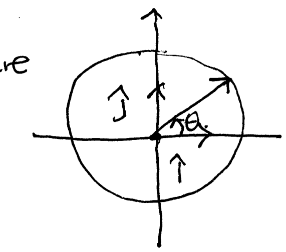
$= \langle a\cos t, a\sin t, bt\rangle$

$\quad + \frac{a^2+b^2}{a}\langle -\cos t, -\sin t, 0\rangle$

$\quad + \frac{a^2+b^2}{a}\left[\cos\theta \frac{1}{\sqrt{a^2+b^2}}\langle -a\sin t, a\cos t, b\rangle + \sin\theta\langle -\cos t, -\sin t, 0\rangle\right]$

no need to combine components to use in technology.

Compare

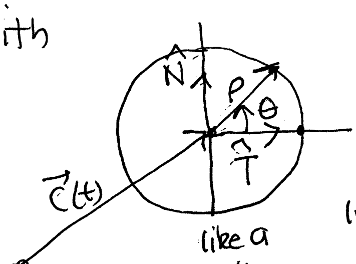


circle at origin of xy plane

$\vec{r}(\theta) = \langle a\cos\theta, a\sin\theta\rangle$

$= \underset{\text{center}}{\vec{0}} + a\underbrace{(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath})}_{\text{diff vector from center}}$

with



like a   like $\hat{\imath}$ like $\hat{\jmath}$

$\vec{R}(t,\theta) = \underset{\text{center}}{\vec{C}(t)} + \rho(t)\underbrace{\left[\cos\theta\,\hat{T}(t) + \sin\theta\,\hat{N}(t)\right]}_{\text{difference vector from center}}$

$\theta = 0..2\pi$ starts at tip of $\hat{T}(t)$ and moves towards $\hat{N}(t)$ around circle at fixed t.

This is how Maple plots the osculating circle.

Choosing a radius a for the cylinder containing the helix and an inclination b picks out a particular helix from this 2-parameter family that we can plot with technology. For each value of the curve parameter t, the interval $0 \le \theta \le 2\pi$ traces out the osculating circle at $\vec{r}(t)$. Animating the plot allows the osculating circle to roll along the helix.