double integral iteration exercise

Consider the quarter moon-like region R between the two circles $x^2 + y^2 - 2x = 3$, $x^2 + y^2 + 2x = 8$ [which of the two regions between the two circles looks like a quarter moon?].

a) Find the center points and radii of these two circles.

b) Find the Cartesian coordinates of their two points of intersection (hint, subtract the two equations to get started).

c) Find the corresponding polar coordinates of these two points, choosing acute angles up to sign.

d) Set up two iteration diagrams which graphically organize the information needed to set up two iterated single double integrals of any function over this region in the compatible orders of integration possible for the various Cartesian and angular variables. Shade in this region by equally spaced linear cross-sections representing the inner integral, terminated by bullet points labeled by the equations stating the starting and stopping values of the variable increasing along that direction, with a middle arrow head emphasizing that increasing direction. In the polar case, label the half lines of the starting and stopping angular variable as well. Make sure you label the intersection points with the appropriate coordinates in each diagram.

e) From these two iteration diagrams, write down the corresponding iterated single double integrals of the constant function 1 over this region to evaluate its area in the compatible orders of integration possible for the various Cartesian and angular variables.

f) Evaluate these integrals using Maple. They should agree and their common numerical value should look right considering you can easily implicit plot the two circles and judge what fraction of one of the circles this represents. Even a hand plot would probably be sufficient. Discuss this estimate to put your number into context.

▶ solution