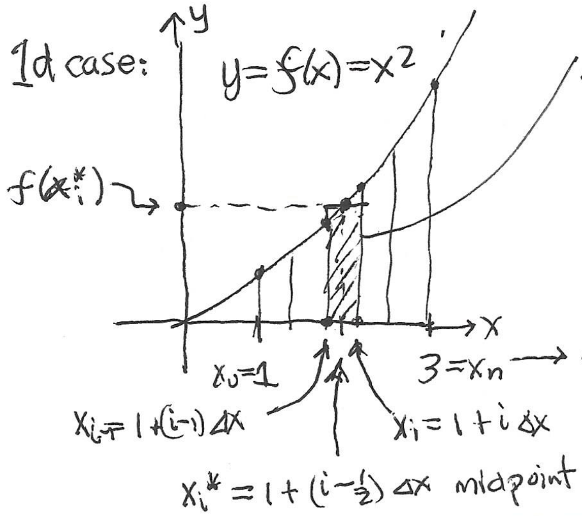
Riemann limit gives volume interpretation to double integral

1d case:



$\Delta x=\frac{3-1}{n}=\frac{2}{n}$

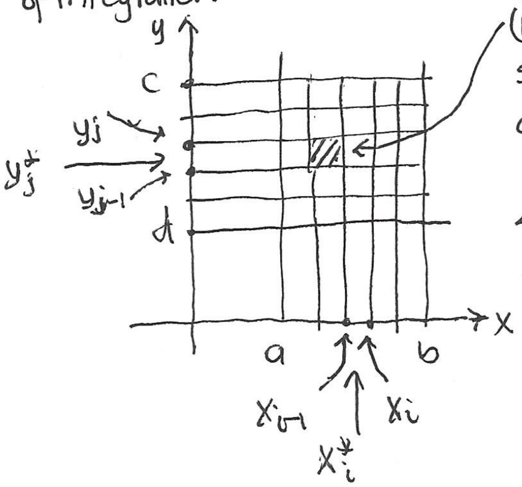
$\Delta A_i=\underbrace{\underbrace{f(x_i^*)}_{\text{sampled value}}\ \underbrace{\Delta x}_{\text{width base}}}_{\text{area rectangle}}$

$\displaystyle\int_1^3 f(x)\,dx=\lim_{n\to\infty}\overbrace{\sum_{i=1}^{n}\overbrace{f(x_i^*)\,\Delta x}^{\Delta A_i}}^{A_n=\text{approx integral}}$

$=$ area under graph

2d case:

rectangular region R: $[a,b]\times[c,d]$ of integration



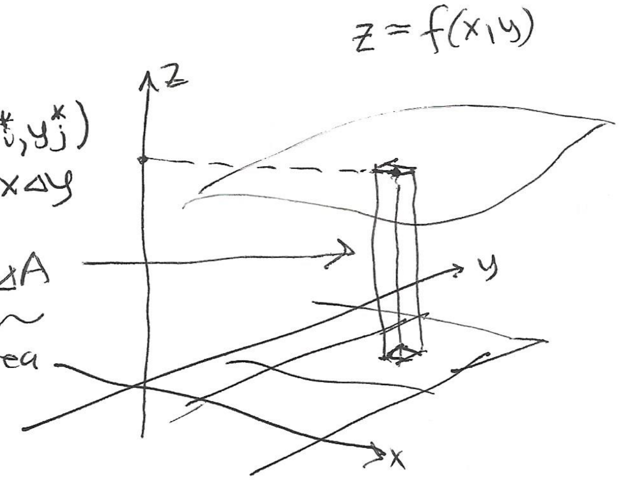
(i,j) box

sampled value: $f(x_i^*,y_j^*)$

area base: $\Delta A=\Delta x\,\Delta y$

$\Delta V_{i,j}=\underbrace{f(x_i^*,y_j^*)}_{\text{height}}\ \underbrace{\Delta A}_{\text{area}}$

volume rectangular box



differential of area:

$\displaystyle\iint_R f(x,y)\,dA \;=\; \underbrace{\lim_{n\to\infty}\lim_{m\to\infty}\underbrace{\sum_{i=1}^{n}\sum_{j=1}^{m} f(x_i^*,y_j^*)\,\Delta x\,\Delta y}_{V_{(n,m)}\ \text{approximate integral}}}_{\text{volume under graph}}$

integral of f w.r.t. area of base

(signed volume when f not nonnegative)

ApproximateIntTutor helps you evaluate approximate integrals