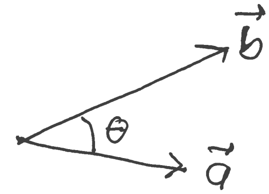
# dot product

geometrical def: $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$




component def:

$\vec{a}\cdot\vec{b} = \langle a_1, a_2, a_3\rangle \cdot \langle b_1, b_2, b_3\rangle$

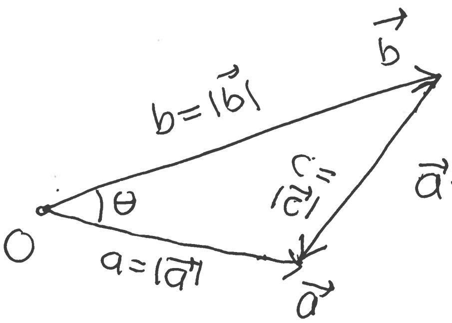
$= a_1b_1 + a_2b_2 + a_3b_3$ ← we compute with this.

we interpret the result with this.

$\vec{a}\cdot\vec{a} = a_1^2 + a_2^2 + a_3^2 = |\vec{a}|^2$

## why are they in agreement?

(use component def here)




$\vec{a} - \vec{b} = \vec{c}$

law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\theta$

$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

$(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b}) = \vec{a}\cdot\vec{a} + \vec{b}\cdot\vec{b} - 2|\vec{a}||\vec{b}|\cos\theta$

multiply out: $\cancel{\vec{a}\cdot\vec{a}} \underbrace{- \vec{b}\cdot\vec{a} - \vec{a}\cdot\vec{b}}_{-2\vec{a}\cdot\vec{b}} + \cancel{\vec{b}\cdot\vec{b}} = \cancel{\vec{a}\cdot\vec{a}} + \cancel{\vec{b}\cdot\vec{b}} - 2|\vec{a}||\vec{b}|\cos\theta$ cancel

so $-2\,\vec{a}\cdot\vec{b} = -2|\vec{a}||\vec{b}|\cos\theta$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ ✓

## one vector:

$|\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}}$

$\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

so $\vec{a} = \langle a_1, a_2, a_3\rangle \leftrightarrow (|\vec{a}|, \hat{a})$

components wrt some Cartesian coord system

length, direction = geometric information

multiply thru → $\vec{a} = |\vec{a}|\,\hat{a}$ decomposes vector into geometrical info

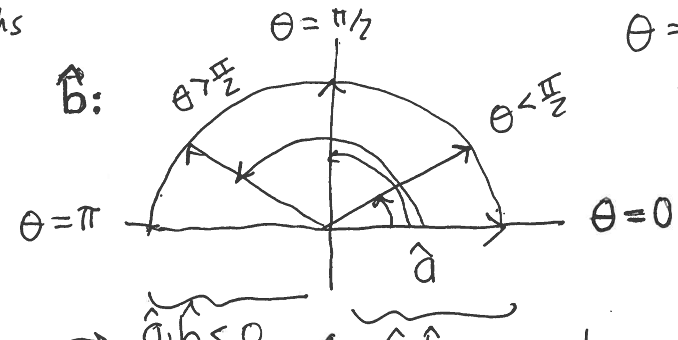
## two vectors:

$\vec{a}, \vec{b} \rightarrow |\vec{a}|, |\vec{b}|$ lengths

$\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{|\vec{a}||\vec{b}|\cos\theta}{|\vec{a}||\vec{b}|} \longrightarrow \cos\theta = \hat{a}\cdot\hat{b}$ angle between

$\downarrow$

$\theta = \arccos(\hat{a}\cdot\hat{b}) \in [0, \pi]$




only angles between $0^\circ$ and $180^\circ$!

$\vec{a}\cdot\vec{b} < 0$ → $\hat{a}\cdot\hat{b} < 0$ obtuse

$\hat{a}\cdot\hat{b} = 0$ orthogonal $\vec{a}\cdot\vec{b} = 0$

$\hat{a}\cdot\hat{b} > 0$ acute ← $\vec{a}\cdot\vec{b} > 0$

[sign of dot product distinguishes acute, right, obtuse angles]