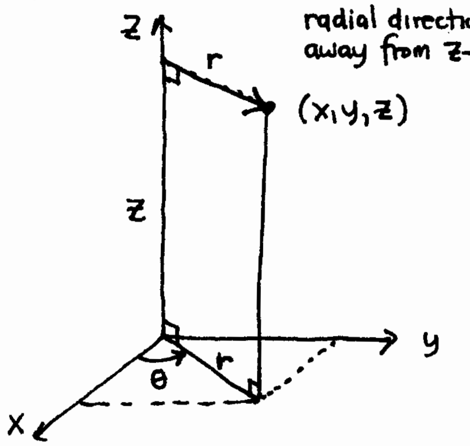
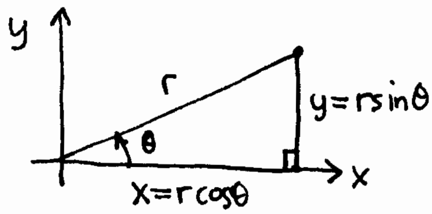
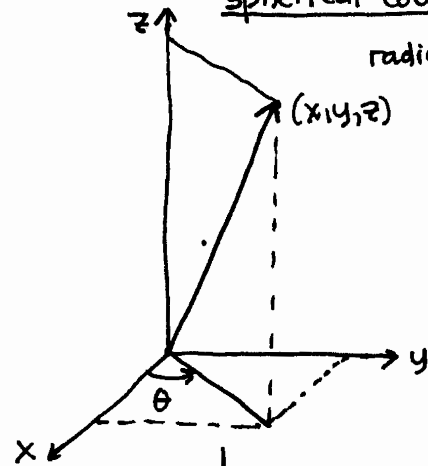
cylindrical coordinates (r, θ, z)



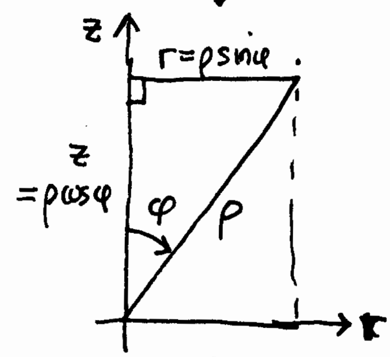


$r \geq 0, \quad 0 \leq \theta \leq 2\pi \quad (\text{or } -\pi \leq \theta \leq \pi)$

spherical coordinates (ρ, φ, θ)



dont memorize
just remember diagram



$$\boxed{\begin{aligned} r &= \rho \sin\varphi \\ z &= \rho\cos\varphi \end{aligned}}$$

$\rho \geq 0, \quad \underset{(\text{up})}{0} \leq \varphi \leq \underset{(\text{down})}{\pi}$

→ keep z, polar coords in xy plane → keep θ, polar coords in rz plane

x	$=$	$r\cos\theta$	$= (\rho\sin\varphi)\cos\theta$
y	$=$	$r\sin\theta$	$= (\rho\sin\varphi)\sin\theta$
z	$=$	z	$= (\rho\cos\varphi)$

$x^2+y^2 = r^2 \to r = \sqrt{x^2+y^2} \geq 0$

$\frac{y}{x} = \tan\theta \to \theta = \arctan\frac{y}{x} + \begin{cases} 0; & \text{I, IV} \\ \pi; & \text{II} \\ -\pi; & \text{III} \end{cases}$ (quad)

$x^2+y^2+z^2 = \rho^2 \to \rho = \sqrt{x^2+y^2+z^2}$
$= \sqrt{r^2+z^2}$

$\cos\varphi = \frac{z}{\rho} = \frac{z}{\sqrt{x^2+y^2+z^2}}$

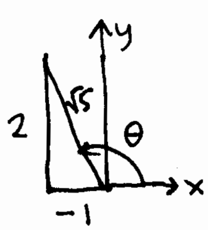
$\varphi = \arccos\left(\frac{z}{\rho}\right) = \arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$

Example. Find cyl/sph coords of $(x,y,z) = (-1,2,1)$.

visualize
think, use simple trig

$(-1, 2, 1) \longrightarrow \rho = \sqrt{1+4+1} = \sqrt{6}$

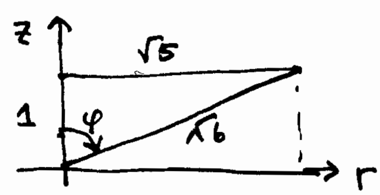
$r = \sqrt{1+4} = \sqrt{5} \to z=1 \longrightarrow$ or $\rho = \sqrt{5+1} = \sqrt{6}$



$\tan\theta = \frac{2}{-1}$

$\theta = \pi - \arctan 2$

$(\approx 116.6°)$



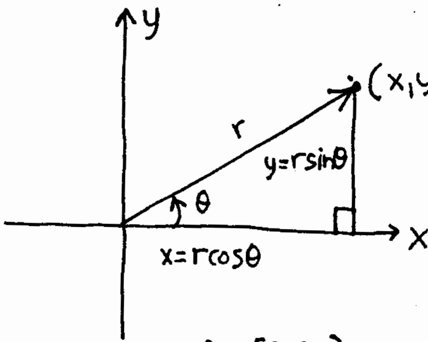
$\cos\varphi = \frac{1}{\sqrt{6}}$

$\varphi = \arccos\frac{1}{\sqrt{6}}$

$(\approx 65.9°)$

polar, cylindrical, spherical coordinates

polar coords in plane



$(x,y) = (r\cos\theta, r\sin\theta)$

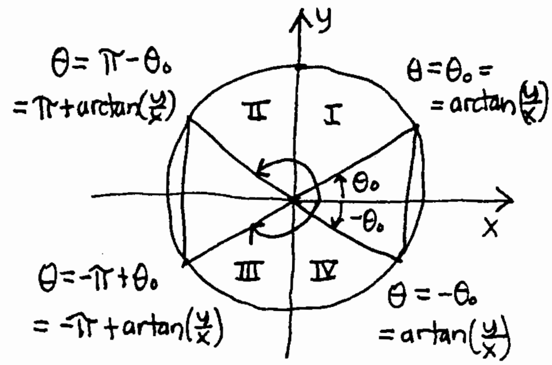
$x = r\cos\theta$
$y = r\sin\theta$

$\Updownarrow$

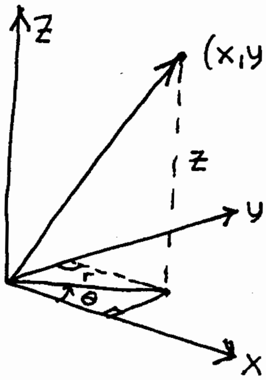
$r = \sqrt{x^2+y^2} \geq 0$

$\theta \in [0, 2\pi)$
or $\in (-\pi, \pi)$ $\rightarrow$ $\tan\theta = \frac{y}{x}$ $\leftarrow$

MAPLE: $\theta = \arctan(y, x) \in (-\pi, \pi]$



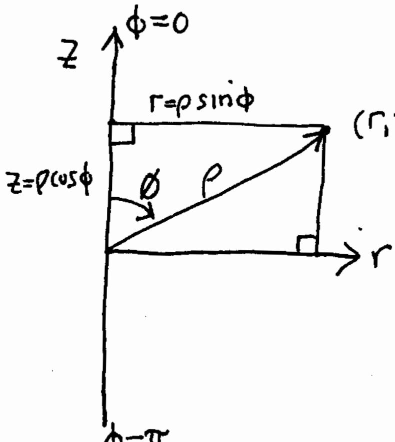
cyl coords



$(x,y,z) = (r\cos\theta, r\sin\theta, z)$

$x = r\cos\theta$ $\qquad$ $r = \sqrt{x^2+y^2}$
$y = r\sin\theta$ $\leftrightarrow$ $\tan\theta = \frac{y}{x}$
$z = z$ $\qquad$ $z = z$

cyl to sphere



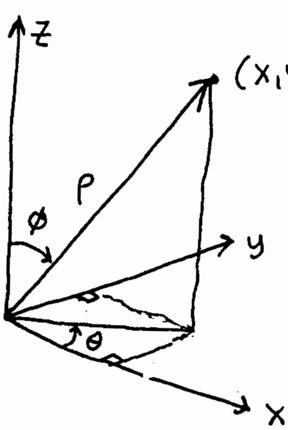
$(r,z) = (\rho\sin\phi, \rho\cos\phi)$

introduce
polar coords in r–z plane
(halfplane $r \geq 0$) of fixed θ
but measured from the z-axis:
$0 \leq \phi \leq \pi$

$r = \rho\sin\phi$
$\theta = \theta$ $\qquad \leftrightarrow \qquad$
$z = \rho\cos\phi$

$\rho = \sqrt{r^2+z^2} \geq 0$
$\theta = \theta$
$$\tan\phi = \frac{r}{z} \leftrightarrow \phi = \begin{cases} \arctan\left(\frac{r}{z}\right) \in [0, \frac{\pi}{2}), & z>0 \\ \pi + \arctan\left(\frac{r}{z}\right) \in (\frac{\pi}{2}, \pi], & z<0 \end{cases}$$
$$= \text{arccot}\left(\frac{z}{r}\right) \in [0, \pi]$$

sph coords



$(x,y,z) = (\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)$

$x = \rho\sin\phi\cos\theta$ $\qquad$ $\rho = \sqrt{x^2+y^2+z^2}$
$y = \rho\sin\phi\sin\theta$ $\leftrightarrow$ $\tan\phi = \sqrt{x^2+y^2}/z$
$z = \rho\cos\phi$ $\qquad$ $\tan\theta = \frac{y}{x}$