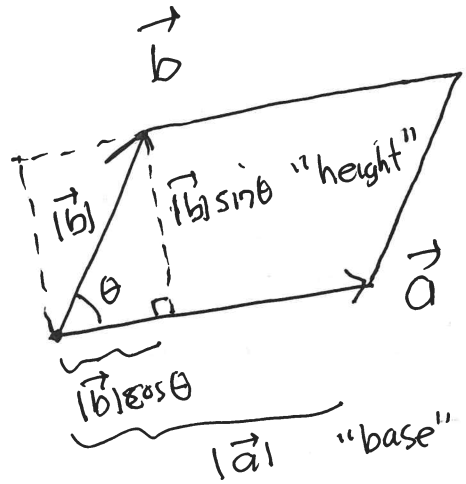
cross product $\vec{a} \times \vec{b}$: geometric definition

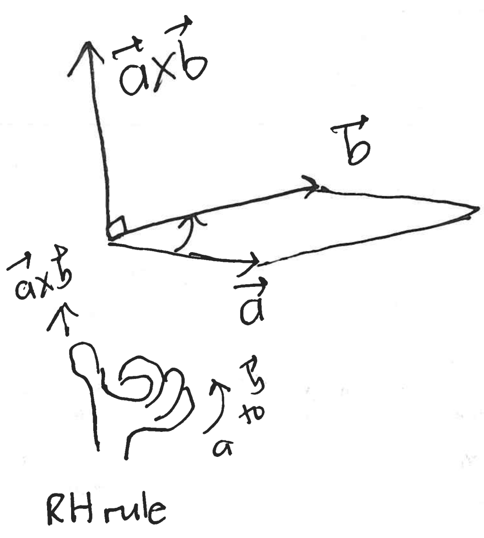


Note;
$0 \le \theta \le \pi$
so $\sin\theta \ge 0$

Any two nonzero noncollinear vectors form a parallelogram whose area is "base" x "height"

$$|\vec{a}| \; |\vec{b}| \sin\theta = \text{area parallelogram} \ge 0$$

$$\equiv |\vec{a} \times \vec{b}|$$ defined to be the length of the cross product vector



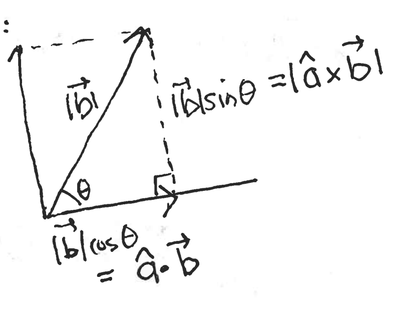
RH rule

The direction of $\vec{a} \times \vec{b}$ is perpendicular to their plane on the side determined by the right hand rule

1) When $\vec{a}, \vec{b}$ are collinear (parallel) the area goes to zero so $|\vec{a} \times \vec{b}| = 0$ which forces $\vec{a} \times \vec{b} = \vec{0}$

2) $\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$ since right hand rule flips the direction

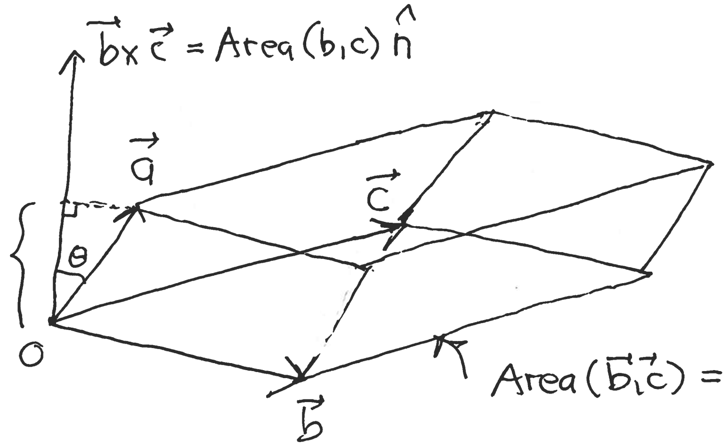
NOTE:



unit vector produced scalars encode cos, sin

triple scalar product

3 noncoplanar vectors determine a parallelopiped



$|\vec{a}|\cos\theta$
$= \hat{n}\cdot\vec{a}$
$= \pm$ "height"
(acute / obtuse angle)

$\text{Area}(\vec{b},\vec{c}) = |\vec{b}\times\vec{c}|$

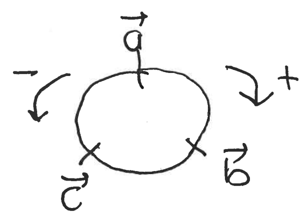
$\vec{b}\times\vec{c} = \text{Area}(b,c)\,\hat{n}$

scalar: $\vec{a}\cdot(\vec{b}\times\vec{c}) = \underbrace{\vec{a}\cdot\hat{n}}_{\pm\text{"height"}}\,\text{Area}(b,c) = \pm\,\text{Volume}(\vec{a},\vec{b},\vec{c})$

so $\boxed{\text{Volume}(\vec{a},\vec{b},\vec{c}) = |\vec{a}\cdot(\vec{b}\times\vec{c})|}$

order unimportant to volume, so all orderings must have same abs value.

Fact: $\vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{b}\cdot(\vec{c}\times\vec{a}) = \vec{c}\cdot(\vec{a}\times\vec{b})$
$= -\vec{a}\cdot(\vec{c}\times\vec{b}) = -\vec{b}\cdot(\vec{a}\times\vec{c}) = -\vec{c}\cdot(\vec{b}\times\vec{a})$



cyclic, anticyclic permutations determine sign.

triple cross product (not necessary for MAT2500)

identity: $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

specialize to: $\hat{a}\times(\hat{a}\times\vec{c}) = (\hat{a}\cdot\vec{c})\,\hat{a} - \underbrace{(\hat{a}\cdot\hat{a})}_{1}\vec{c}$

$$= -(\vec{c} - \underbrace{(\vec{c}\cdot\hat{a})\hat{a}}_{\vec{c}_{\parallel}}) = -\vec{c}_{\perp}$$

so $\vec{c}_{\perp} = -\hat{a}\times(\hat{a}\times\vec{c})$

orthogonal projection wrt $\hat{a}$ of $\vec{c}$

$\vec{c} = \vec{c}_{\parallel} + \vec{c}_{\perp}$