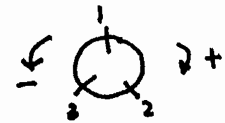
Brute force computation: Magnitude of cross product



$$\vec a \times \vec b = \langle \underset{123}{a_2b_3 - a_3b_2},\ \underset{231}{a_3b_1 - a_1b_3},\ \underset{312}{a_1b_2 - a_2b_1} \rangle$$

$$\Rightarrow |\vec a \times \vec b|^2 = \begin{array}{l} (a_2b_3 - a_3b_2)^2 \\ +(a_3b_1 - a_1b_3)^2 \\ +(a_1b_2 - a_2b_1)^2 \end{array} = \begin{array}{l} a_2^2b_3^2 + a_3^2b_2^2 - 2a_2a_3b_2b_3 \\ +a_3^2b_1^2 + a_1^2b_3^2 - 2a_3a_1b_3b_1 \\ +a_1^2b_2^2 + a_2^2b_1^2 - 2a_1a_2b_1b_2 \end{array}$$

equal!! (compare 9 terms)

compare:

$$|\vec a|^2|\vec b|^2 - (a\cdot b)^2 = (a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2) - (a_1b_1 + a_2b_2 + a_3b_3)^2 = \begin{array}{l} a_1^2b_1^2 + a_1^2b_2^2 + a_1^2b_3^2 \\ +a_2^2b_1^2 + a_2^2b_2^2 + a_2^2b_3^2 \\ +a_3^2b_1^2 + a_3^2b_2^2 + a_3^2b_3^2 \end{array} - \left[\begin{array}{l} a_1^2b_1^2 + 2a_1a_2b_1b_2 \\ +a_2^2b_2^2 + 2a_2a_3b_2b_3 \\ +a_3^2b_3^2 + 2a_3a_1b_3b_1 \end{array}\right]$$

0

cancel

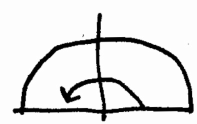
$|a|^2|b|^2\cos^2\theta$

$$\| $$

$$|\vec a|^2|\vec b|^2 - |\vec a|^2|\vec b|^2\cos^2\theta$$

$$\| $$

$$|\vec a|^2|\vec b|^2(1-\cos^2\theta) = |\vec a|^2|\vec b|^2\sin^2\theta \quad \blacksquare$$



$0 \le \theta \le \pi$

$\sin\theta \ge 0$

$\sqrt{\sin^2\theta} = \sin\theta$

$$\therefore \mapsto \quad |\vec a \times \vec b| = |\vec a||\vec b|\sin\theta$$

RIGHT HAND RULE WORKS ON BASIC UNIT VECTORS ALONG AXES

$$\hat\imath \times \hat\jmath = \langle \underset{a_1}{1},0,0\rangle \times \langle 0,\underset{b_2}{1},0\rangle = \langle 0,0,\underset{a_1b_2}{1}\rangle = \hat k$$

$$\hat\jmath \times \hat k = \langle 0,\underset{a_2}{1},0\rangle \times \langle 0,0,\underset{b_3}{1}\rangle = \langle \underset{a_2b_3}{1},0,0\rangle = \hat\imath$$

$$\hat k \times \hat\imath = \langle 0,0,\underset{a_3}{1}\rangle \times \langle \underset{b_1}{1},0,0\rangle = \langle 0,\underset{a_3b_1}{1},0\rangle = \hat\jmath$$