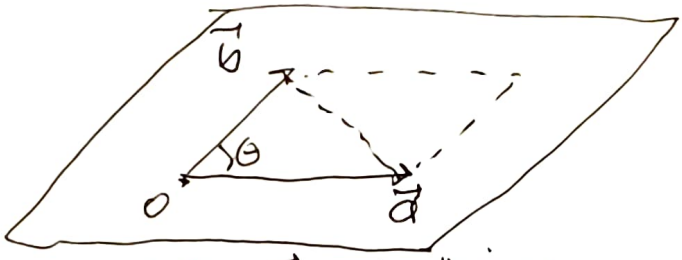
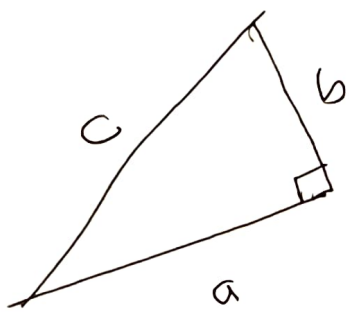
Plane vectors and trig



$\vec{a}, \vec{b} \neq \vec{0}$ not collinear (not parallel)

In any dimension, any 2 vectors determine a plane which contains them as well as a triangle with an included angle and a parallelogram.

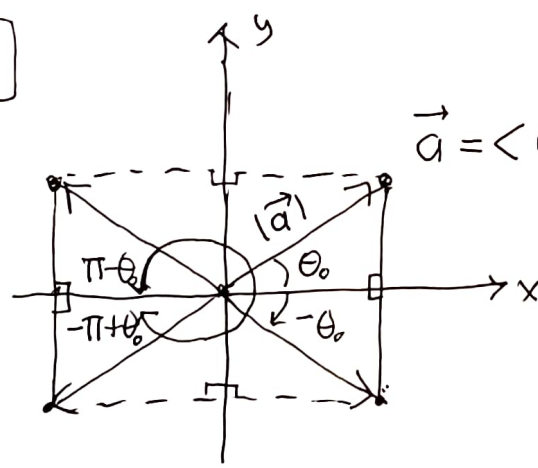
Ordinary trigonometry holds within that plane.
The vector multiplication operations of dot & cross products generalize this to any orientation within space.



$\boxed{\text{Trigonometry}}$ is the geometry of the right triangle, based on the Pythagorean theorem.

The "$\cdot$" and "$\times$" products generalize the trigonometry of the plane to space.

$\boxed{\text{vectors in the plane}}$



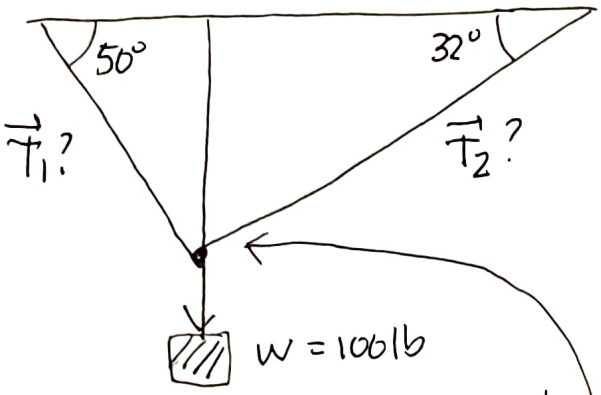
$$\vec{a} = \langle a_1, a_2 \rangle = \underbrace{|\vec{a}|}_{\text{magnitude}} \underbrace{\langle \cos\theta, \sin\theta \rangle}_{\hat{a}\ \text{direction}}$$

The components of a vector are obtained by "projecting" the vector onto the coordinate axes

An angle $-\pi \leq \theta < \pi$ determines its direction ← PREFERRED
(sometimes $0 \leq \theta < 2\pi$) ← sometimes convenient.

12.2b Plane vectors and trig

Example word problem:



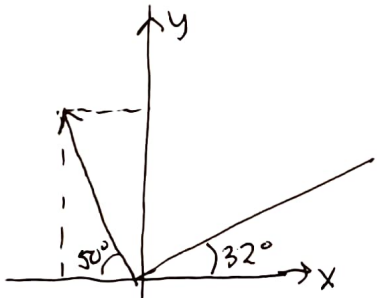
not yet a math diagram!
put origin of x-y axes here

A 100lb weight hangs from 2 wires as shown.

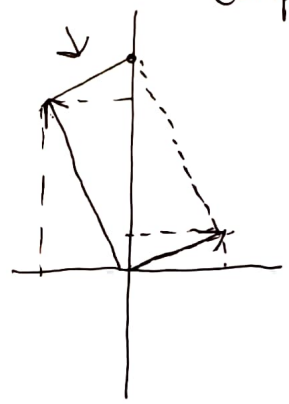
Find the tensions $\vec{T}_1$ and $\vec{T}_2$ (forces)

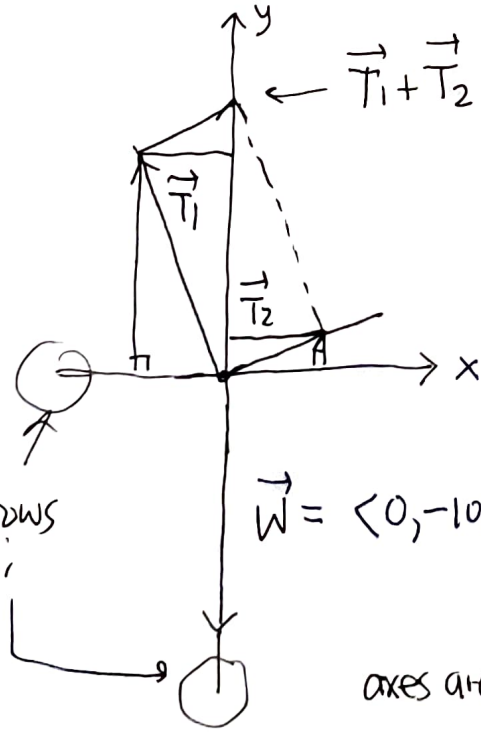
boldface in book
over arrows ALWAYS
in hand notation

complete to a parallelogram with vertex on y axis



step one





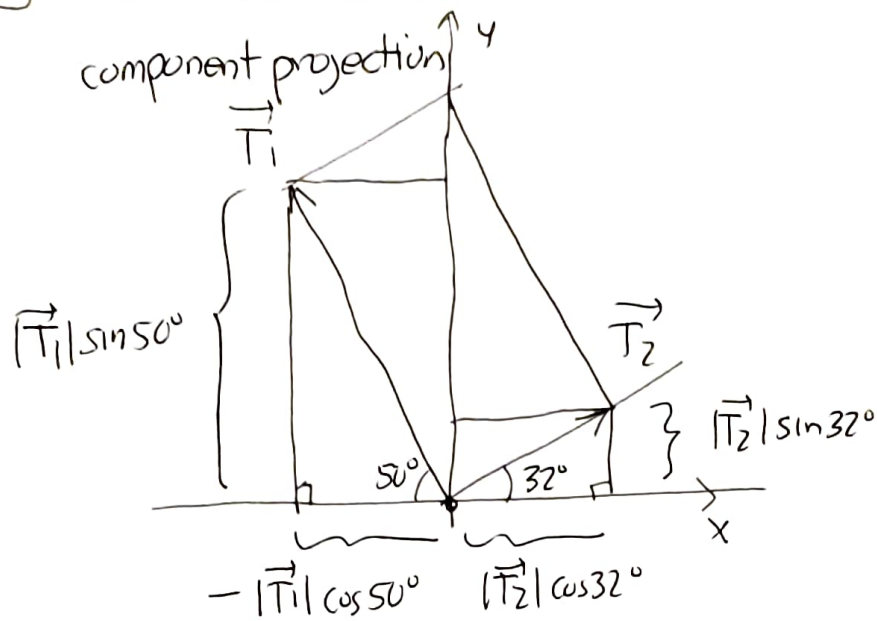
$\vec{T}_1 + \vec{T}_2 = -\vec{W}$ force balance

$\vec{W} = \langle 0, -100 \rangle$

no arrows here,

axes are "oriented" with increasing direction

Plane vectors and trig



$$\vec{T_1} = \langle -|\vec{T_1}|\cos 50°,\ |\vec{T_1}|\sin 50° \rangle$$

$$\vec{T_2} = \langle |\vec{T_2}|\cos 32°,\ |\vec{T_2}|\sin 32° \rangle$$

$$\vec{T_1}+\vec{T_2} = \langle -|\vec{T_1}|\cos 50° + |\vec{T_2}|\cos 32°,\ |\vec{T_1}|\sin 50° + |\vec{T_2}|\sin 32° \rangle$$

$$= -\vec{W} = \langle 0,\ 100 \rangle$$

sets the overall scale of the problem

$$-|\vec{T_1}|\cos 50° + |\vec{T_2}|\cos 32° = 0 \rightarrow |\vec{T_2}| = |\vec{T_1}|\frac{\cos 50°}{\cos 32°}$$

$$|\vec{T_1}|\sin 50° + |\vec{T_2}|\sin 32° = 100$$

$$|\vec{T_1}|\sin 50° + |\vec{T_1}|\frac{\cos 50°}{\cos 32°}\sin 32° = 100$$

(pen fading contrast messes up scan!)

$$|\vec{T_1}|\left(\sin 50° + \cos 50° \tan 32°\right) = 100$$

$$|\vec{T_1}| = \frac{100}{\sin 50° + \cos 50° \tan 32°}$$

$$|\vec{T_2}| = |\vec{T_1}|\frac{\cos 50°}{\cos 32°}$$

use technology to evaluate final numbers [see Maple]

Remember angles in math are assumed to be in radians NOT DEGREES!
Convert in Maple, multiply by $\frac{\pi \text{ rad}}{180°}$ (neither is a "physical" unit — these are pure numbers!)

$W=100$, $\alpha=50°$, $\beta=32°$
act like placeholders for arbitrary $\boxed{\text{parameter}}$ values.

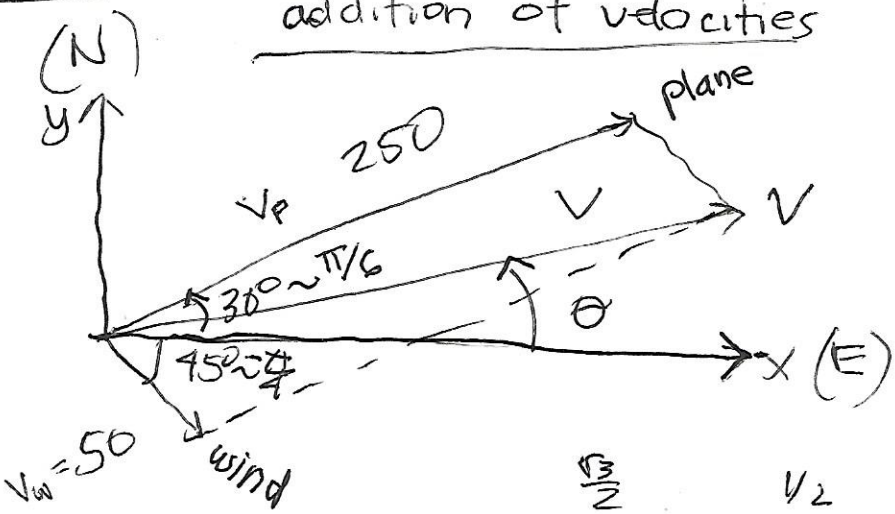
DO NOT plug in numerical values for any of these numbers in your formulas UNTIL final evaluation.

The formulas solve $\boxed{\text{ALL}}$ possible problems of this type for allowable values of α, β, W

For $\boxed{\text{numerical final results}}$ to a certain $\boxed{\text{precision}}$:
n significant digits or n decimal places
use maximum default # digits on your technology device for all intermediate numerical evaluations,
round off final number to desired accuracy.

12.2b plane vectors and trig

addition of velocities



Given plane and wind speeds & angle directions find ground speed and direction

$$V_p = 250 \langle \underbrace{\cos \tfrac{\pi}{6}}_{\frac{\sqrt{3}}{2}}, \underbrace{\sin \tfrac{\pi}{6}}_{1/2} \rangle = 250 \langle \tfrac{\sqrt{3}}{2}, \tfrac{1}{2} \rangle$$

$$V_w = 50 \langle \underbrace{\cos \tfrac{\pi}{4}}_{1/\sqrt{2}}, \underbrace{-\sin \tfrac{\pi}{4}}_{1/\sqrt{2}} \rangle = 50 \langle \tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}} \rangle$$

$$V = V_p + W_w = \langle 250\tfrac{\sqrt{3}}{2}, 250\tfrac{1}{2} \rangle + \langle \tfrac{50}{\sqrt{2}}, -\tfrac{50}{\sqrt{2}} \rangle$$

$$= \langle \tfrac{250\sqrt{3}}{2} + \tfrac{50}{\sqrt{2}}, \tfrac{250}{2} - \tfrac{50}{\sqrt{2}} \rangle$$ no approximate

$$\approx \langle 251.86, 89.645 \rangle \approx \langle 251.9, 89.6 \rangle$$

ground speed $|V| = \sqrt{V_1^2 + V_2^2} \overset{\text{tech}}{=} 267.32 \approx 267.3$

$$\tan\theta = \frac{V_2}{V_1} \rightarrow \theta = \arctan \frac{V_2}{V_1} \overset{\text{tech}}{\approx} 0.3429$$

$$\approx 19.6°$$

The # of significant figures we keep depends on how serious we'ok take the numbers in the problem. We shouldn't keep too many more. We could probably round off to the nearest mph and degree here.