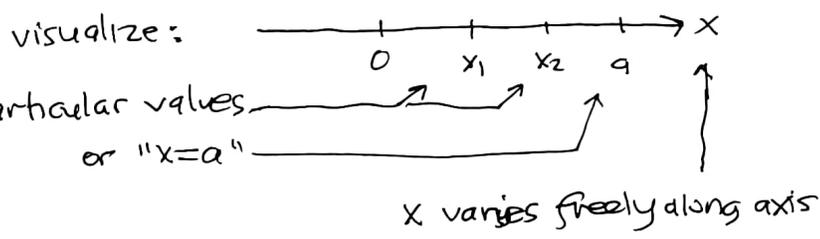


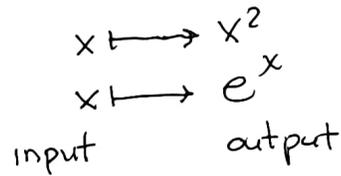
**12.1** 3-d coordinate systems, distance formula, spheres, etc (course intro) ①

Calculus of a single <sup>real!</sup> variable: Notation

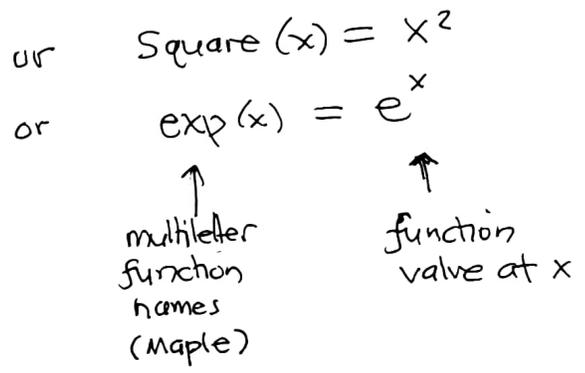
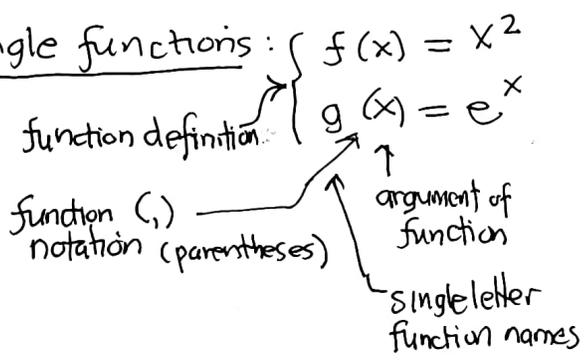
default variable name "x"  
"independent variable"  
subscripts often indicate particular values



single functions (no parameters):



named single functions:



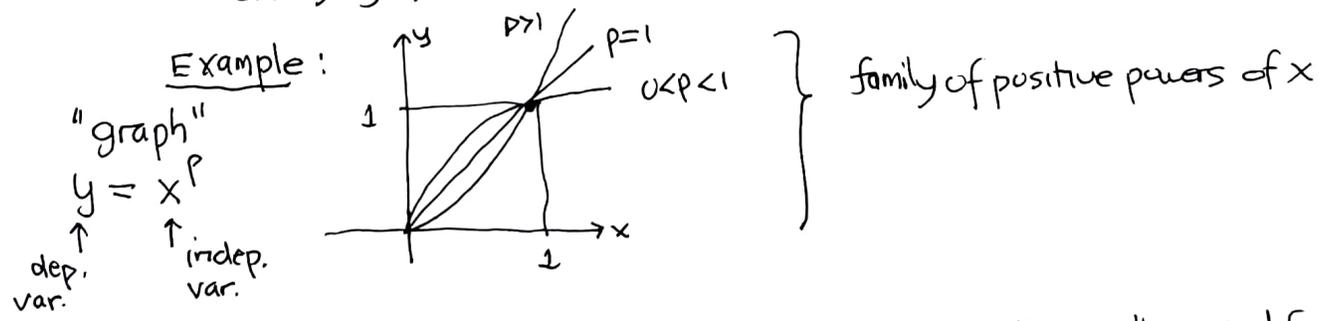
families of functions (1 parameter)

(2 parameters)

power functions  $f(x) = x^p, p \in \mathbb{R}$   
exponential functions  $g(x) = a^x, a > 0$

or  $f(x) = Cx^p$   
or  $g(x) = Ca^x$

parameters are held fixed as we "vary" the "variable"  $x$  to explore the changing function values



"functional relationship"

no named function here!

[In applications there are "never" named functions] only relationships between physical variables.

This is an explicit functional relationship, as opposed to an implicit functional relationship like  $x^2 + y^2 = 1$  implies "locally"  $y$  is some function of  $x$

12.1 3-d coordinate systems, distance formulas, spheres, etc (course intro) (2)

Calculus of  $n > 1$  independent real variables

Examine function values as all "independent" variables vary

single multivariable function:

$$f(x, y) = X^y, x > 0, y \in \mathbb{R}$$

function definition

parenthesis function notation

2 independent variables

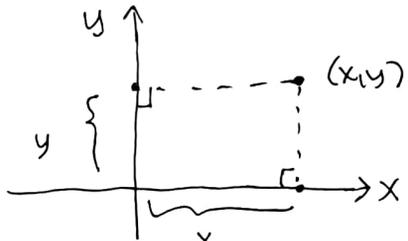
family:

$$g(x, y) = a X^y, a \in \mathbb{R}$$

Function Graphs

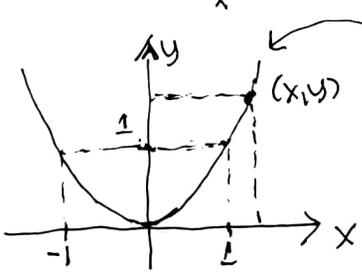
We introduce an extra "dependent" variable to "visualize" a function with a graph.

- 1 ind. var.  $x$
- 1 dep. var.  $y$



rectangular coordinate system ("Cartesian")

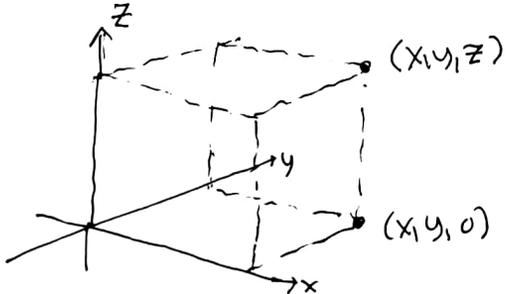
function graph (curve in xy plane)



$y = x^2$   
 ↑ ind. var.  
 ↑ dep. var.

explicit functional relationship  
 only one value of  $y$  for each value of  $x$  ("vertical line test")

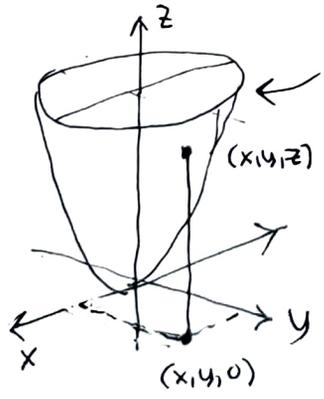
- 2 ind. vars.  $(x, y)$
- 1 dep. var.  $z$



rectangular box locates each point

only one value of  $z$  for each pair  $(x, y)$

function graph (surface in xy-z space)



$$z = f(x, y) = x^2 + y^2$$

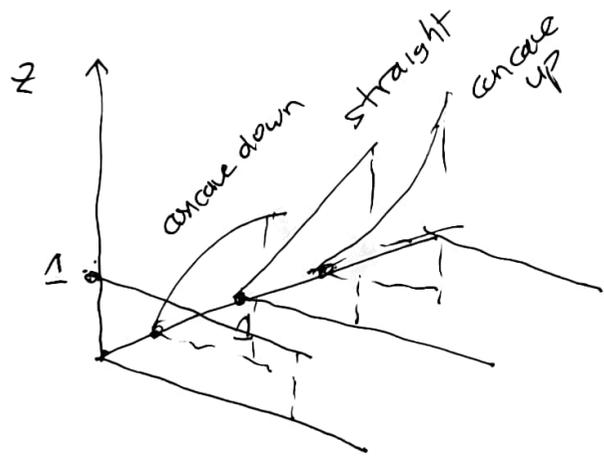
12.1 3-d coordinate systems, distance formula, spheres, etc (course intro) 3

Example: graph  $z = x^y$  ??  $x > 0, y \geq 0$  for simplicity

for fixed  $x$  like an exponential function  
all increasing functions

for fixed  $y$  like a power function  
 $y > 1$  concave up  
 $y = 1$  straight line  
 $0 < y < 1$  concave down  
 $y = 0 : x^0 = 1$  ( $x \neq 0!$ )

↓ draw?



but draw?  
nope  
we need  
technology!

[Maple in a bit]

Basic operations for calculus of a single (real) variable

lim, diff, int (limits, differentiation, integration)

These single variable operations can all be applied to a multivariable function with respect to each of its independent variables in turn, holding the remaining variables fixed as though they were just parameters.

example  $x > 0$ :  $\lim_{y \rightarrow 0} x^y = x^0 = 1$  no problem at  $y=0$ , just plug in!

$\rightarrow \frac{d}{dx} x^y = y x^{y-1} (= \frac{y}{x} x^y)$  power rule EASY!

$\rightarrow \frac{d}{dy} x^y = \frac{d}{dy} ((e^{\ln x})^y) = \frac{d}{dy} e^{y \ln x} = \underbrace{e^{y \ln x}}_{x^y} \underbrace{\frac{d}{dy} (y \ln x)}_{\ln x} = (\ln x) x^y$  EASY!

"partial derivatives"

new notation:  $\frac{d}{dx} \rightarrow \frac{\partial}{\partial x}$

$\frac{d}{dy} \rightarrow \frac{\partial}{\partial y}$

$\rightarrow \int x^y dx = \frac{x^{y+1}}{y+1} + C$  power rule

$\rightarrow \int x^y dy = \int e^{y \ln x} dy = \frac{e^{y \ln x}}{\ln x} + C = \frac{x^y}{\ln x} + C$

"partial integrals"

$u = y \ln x$   
 "u-substitution"  
 $du = (\ln x) dy$  (details not shown!)

Q: So we can use all of our single variable calc operations in multivariable calculus so what do we need to study here?

A: We need to interpret these operations in the context of more variables, to visualize what they represent. AND PACKAGE them as vector operations.

12.1 3-d rectangular coords, distance, spheres, etc (INTRO) (5)

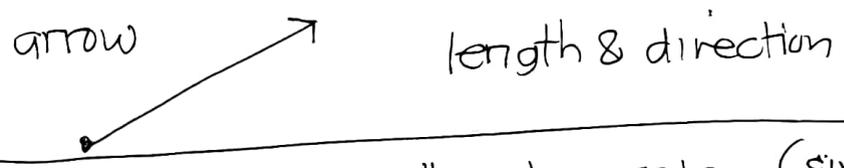
single numbers:  $x, y, z, 1$  are called scalars  
we put them together to make vectors  
so we can simultaneously apply "scalar" operations to each component of the vector.

example:  $2 \langle x^2y, xy^3, x+y \rangle = \langle 2x^2y, 2xy^3, 2x+2y \rangle$   
 $\frac{d}{dx} \langle x^2y, xy^3, x+y \rangle = \langle 2xy, y^3, 1 \rangle$   
etc.

vectors organize our scalar variable information.

$\vec{X} = \langle x, y, z \rangle$  is a vector variable  
 $\uparrow \uparrow \uparrow$  scalar variables

vectors help visualize collections of numbers:



Multivariable calc "vectorizes" scalar calc (single variable)

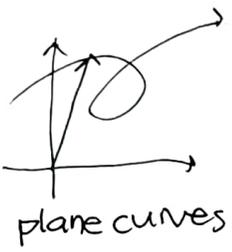
# overview of semester

CHAPTER 12

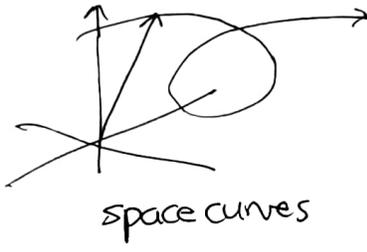
vector algebra and geometry: "•" "x"

CHAPTER 13

vector calculus: 1 ind. var., multiple dep. vars.



plane curves



space curves

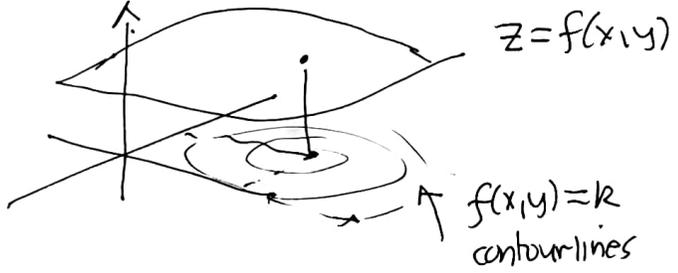
$$\vec{F}(t) = \langle F_1(t), F_2(t), F_3(t) \rangle$$

vector-valued functions of 1 variable

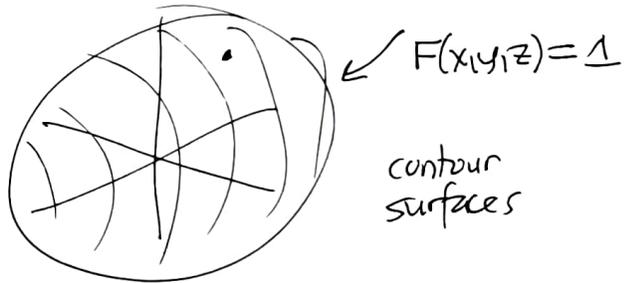
visualization of vector-valued functions tangent lines, etc.

CHAPTER 14

"multivariate" calculus: 1 dep. var., multiple ind. vars.



f(x, y) = k contour lines

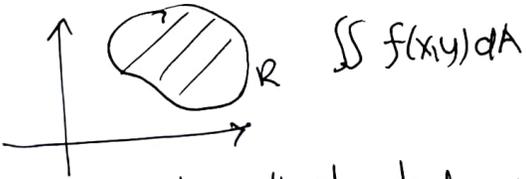


contour surfaces

tangent planes, etc.

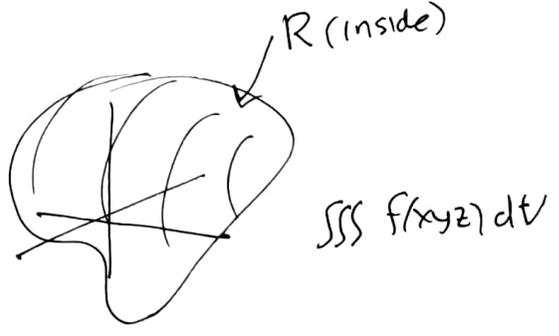
CHAPTER 15

multivariable integration



$$\iint f(x, y) dA$$

evaluated with iterated partial integrations



$$\iiint f(x, y, z) dV$$

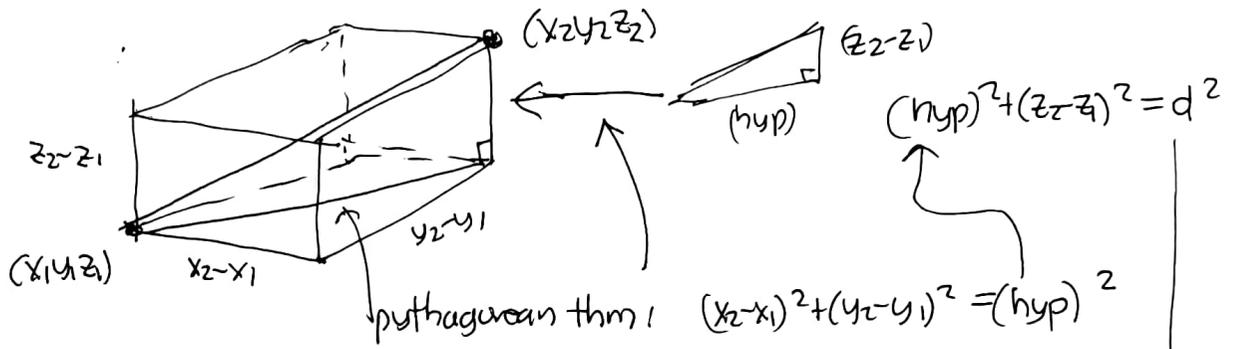
CHAPTER 16

vector field calculus: differentiation & integration dep vector variable & ind. vector variable have same dimension

$$\vec{F} = \langle F_1(x, y), F_2(x, y) \rangle \quad \text{or} \quad \vec{F} = \langle F_1(x, y, z), F_2(x, y, z), F_3(x, y, z) \rangle$$

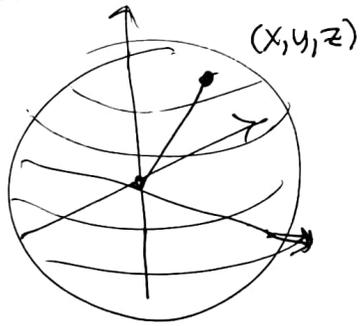
Review section 12.1: rectangular coords and:

lines, planes, curves, surfaces in 2-d or 3-d  
distance formula

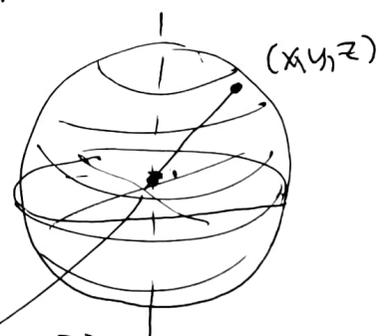


$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 = d^2$$

spheres



$x^2 + y^2 + z^2 = r^2$   
 at origin



$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$

complete square to put into standard form and identify center and radius  
 ↓ multiply out  
 $x^2 + y^2 + z^2 + ax + by + cz = d$