

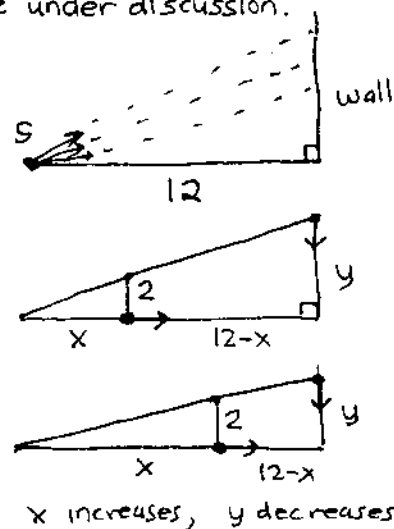
The Long Story: translating a related rates word problem into
 1) the rate you know, 2) the rate you want to know, and 3) the relationship between the underlying variables whose rates of change are under discussion.

A spotlight on the ground shines on a wall **12 m** away,

If a man **2 m tall** walks from the spotlight toward the building at a **speed of 1.6 m/s**

how fast is the **length of his shadow** on the building decreasing

when he is **4 m from the building**?



The man's distance from the spotlight increases at a given rate, so we need to introduce a variable x representing that changing distance so its derivative will be the given rate of change:

$$\frac{dx}{dt} = 1.6 \left(\frac{m}{s} \right)$$

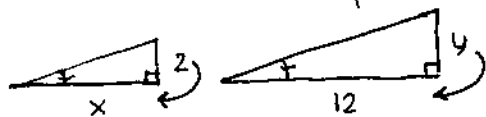
We also need information about the remaining distance to the wall which is $12-x$, so we put that in the diagram too.

We want to calculate the rate of change of the changing length of the shadow, so we must introduce another variable y to represent that changing length so that $\frac{dy}{dt}$ is the rate we want to calculate. We put y in the diagram.

The word "when" signals what values for the changing variables we will use when we evaluate their rates of change: when $12-x = 4 \rightarrow x = 12-4 = 8$, so the rate we want to calculate is

$$\frac{dy}{dt} \Big|_{x=8} = ?$$

Finally we need to establish a relationship between the changing values of x and y so that its derivative will provide a relationship between their rates of change.



similar triangles: corresponding side ratios are equal

$$\frac{y}{12} = \frac{2}{x} \rightarrow y = \frac{24}{x}$$

so $y = \frac{24}{x}$, $\frac{dx}{dt} = 1.6$, $\frac{dy}{dt} \Big|_{x=8} = ?$

are the two statements and question in mathematical language.

$$\frac{d}{dt} \left[y = \frac{24}{x} \right] \rightarrow \frac{dy}{dt} = \frac{d}{dt} (24x^{-1}) = 24(-x^{-2}) \frac{dx}{dt} = -\frac{24}{x^2} \frac{dx}{dt} \rightarrow \frac{dy}{dt} = \frac{-24}{x^2} \frac{dx}{dt} = \frac{-24(1.6)}{x^2}$$

variable relationship.

$$\frac{dy}{dt} \Big|_{x=8} = -\frac{24}{8^2} (1.6) = \dots = \boxed{-0.6 \frac{m}{s}} \quad (\text{value at a particular moment})$$

The word "decreasing" signals us to remove the minus sign from our word answer:

The length of the shadow is decreasing at 0.6 m/s when the man is 4m from the building.