TEST 1 FINAL COMMENT

The final point I wanted to make on asymptotes but for which no time remained was the following:

Suppose instead of a 1 in the factored formula for problem 4 we had a parameter a:

$$f(x) = \frac{2x^2 - 2(a+2)x + 4a}{x^3 - 6x^2 + 12x - 8} = \frac{2(x-a)(x-2)}{(x-2)^3} = \frac{2(x-a)}{(x-2)^2}$$

For f to have a vertical asymptote at x=2, it is enough that the numerator not be zero so first assume $a \neq 2$:

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{2(x-a) \to 2(2-a)}{(x-2)^2 \to 0} \begin{cases} = +\infty \\ = -\infty \end{cases}$$

$2(2-a)$ ← sign depends on a: If $a<2$, then positive, so the whole quotient is positive. If $a>2$, then negative, so the whole quotient is negative.

$(x-2)^2 \to 0$ but always positive because of square

$$\lim_{x\to 2^-} f(x) \;\; // \;\; \text{(so } \lim_{x\to 2^-} \text{ same)}$$

Here we can no longer graph the formula with technology because we don't have a definite value of a, but we can do the simple reasoning that EXPLAINS what we see when a=1.

Even if we tried plotting this for a set of values of a, we would need to think wisely about which values to plot and what we could conclude from those special values. It is simpler to reason directly with the signs of the factors.

Finally suppose a=2:

$$f(x) = \frac{2(x-2)(x-2)}{(x-2)^3} = \frac{2}{x-2} \quad \begin{matrix} \xrightarrow{x\to 2^+} +\infty \\ \xrightarrow{x\to 2^-} -\infty \end{matrix}$$

now the sign of x−2 determines which ∞ we approach on either side, so we get a completely different behavior at the vertical asymptote.

We are trying to develop some elementary math reasoning skills so we can explain what we see in technology output. If we can do this for concrete functions with no parameters, we can then do it for functions involving parameters where numerics & graphics do not immediately apply. We have therefore broadened our analytical skills.