ratio limits

$$\lim_{x\to a} \frac{f(x) \to c_1 \neq 0 \text{ or } 0}{g(x) \to c_2 \neq 0 \text{ or } 0} \Bigg\} \text{ 4 cases: } \underbrace{\frac{c_1}{c_2},\ \frac{0}{c_2}=0}_{\substack{\text{limit = value} \\ \text{(no problem)}}},\ \frac{c_1}{0},\ \frac{0}{0}$$

separately evaluate top & bot values or limits

"reasoning" for $\frac{c_1 \neq 0}{0}$ limits

let 0^+ = abbreviation for approaching 0 thru positive values
let 0^- = abbreviation for approaching 0 thru negative values

$\frac{c_1}{0^+} = \text{sgn}(c_1)\,\infty$ as $x \to a$, $g(x) \to 0$ thru positive values, so overall sign is that of c_1

$\frac{c_1}{0^-} = -\text{sgn}(c_1)\,\infty$ as $x \to a$, $g(x) \to 0$ thru negative values, so overall sign is opposite to that of c_1

evaluation for $\frac{0}{0}$ limits

Must do algebra manipulation to remove problem if possible, reducing limit to one of the previous cases by:

1) factor, cancel, continue
2) expand (multiply out), cancel then
3) if difference of sqrts, multiply by "conjugate" sum (top & bot), combine, cancel, continue
4) if exp, ln, trig functions involved, use special identities to manipulate expression

ALL OF THIS ALSO APPLIES TO LIMITS AS $x \to \pm\infty$.

Arithmetic with infinity

some symbolic arithmetic with ∞ (treating it like a number) is possible with ∞ (understood to really mean positive ∞ here), some is not

OKAY: $\infty \cdot \infty = \infty$, $\infty + \infty = \infty$, $\frac{c}{\infty} = 0$
$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$, $\ln 0^+ = -\infty$, $e^{\infty} = \infty$
$\ln \infty = \infty$
} unambiguous limits

NOT OKAY: $\frac{\infty}{\infty} \neq 1$, $\infty - \infty \neq 0$ ambiguous limits, due to competition: any outcome is possible