linear approximation versus differential approximation

Approximate $\sin 29° = \sin\left(\frac{29\pi}{180}\right)$.

$29°$ is close to $30° = \frac{30\cdot\pi}{180} = \frac{\pi}{6}$ where we know the sine and cosine exactly, so either approximation should use $30°$ as the reference point

linear approximation

$y = f(x) = \sin x$

$f'(x) = \cos x$

at $x = \pi/6$:

$y = f\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

$\underbrace{f'\left(\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}}$

pt. $\left(\frac{\pi}{6}, \frac{1}{2}\right)$, slope $\frac{\sqrt{3}}{2}$

write eq. of tan line:

$$y - \frac{1}{2} = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$$

solve:

$$y = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$$

$$L(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) \quad \text{linear approximation}$$

now at $x = 29° = \frac{29\cdot\pi}{180}$:

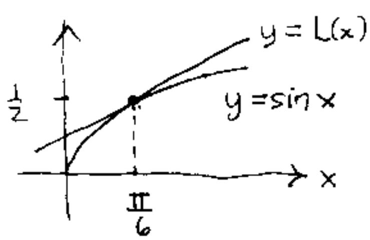
$$f(29°) \approx L(29°) = \frac{1}{2} + \frac{\sqrt{3}}{2}\underbrace{\left(\frac{29\cdot\pi}{180} - \frac{30\cdot\pi}{180}\right)}_{-\frac{\pi}{180} \ \leftarrow \text{ minus 1 degree}}$$

$$= 0.50000 - 0.01511$$

$$= 0.48489 \quad \boxed{\text{done}}$$

compare with:

$$f(29°) = \sin 29° = 0.48481$$



clearly linear approximation is too high

> remember, derivative formulas for trig functions assume angles are given in radians. calculation must be done in radians.

differential approximation

step 1: $y = f(x) = \sin x$

$$\frac{dy}{dx} = f'(x) = \cos x$$

$$dy = f'(x)\,dx = \cos x\,dx$$

at $x = \pi/6$:

$$dy = \cos\frac{\pi}{6}\,dx = \frac{\sqrt{3}}{2}\,dx$$

$$dx = \underset{\text{new}}{29°} - \underset{\substack{\text{old}\\\text{reference}\\\text{value}}}{30°} = -1° = -\frac{1\cdot\pi}{180}$$

then $dy = \frac{\sqrt{3}}{2}\left(\frac{-\pi}{180}\right) = -0.01511$

step 2: at $x = \pi/6$:

$$y = f(\pi/6) = \sin\frac{\pi}{6} = \frac{1}{2}$$

$$= 0.50000$$

new value of y = old value plus change:

$$y + dy = 0.50000 - 0.01511$$

$$= 0.48489 \quad \boxed{\text{done}}$$

compare with:

$$\underset{\text{change}}{\Delta y} = \underset{\text{new}}{\sin 29°} - \underset{\text{old}}{\sin 30°}$$

$$= -0.01519$$

$$dy = -0.01511$$

the differential approximation is too high (less negative)