Icons for how signs of f', f'' affect graph of f

1st derivative

- $f' > 0$: inc.
- $f' = 0$: hor.
- $f' < 0$: dec.

2nd derivative

- $f'' > 0$: curve up
- $f'' < 0$: curve down

combined together

	$f'' > 0$	$f'' < 0$
$f' > 0$	inc, curve up	inc, curve down
$f' < 0$	dec, curve up	dec, curve down

local extrema

at critical pt where $f' = 0$

1st der test

f': $-\ 0\ +$ local min

f': $+\ 0\ -$ local max

sign of f' changes at critical pt

2nd der test

$f'' > 0$

$f'' < 0$

at critical pt where $f' \to \pm\infty$

cusp

f': $-\ \pm\infty\ +$ local min

f': $+\ \pm\infty\ -$ local max

(f'': $-\ \pm\infty\ -$ f'': $+\ \pm\infty\ +$

sign of f'' cannot change across a cusp)

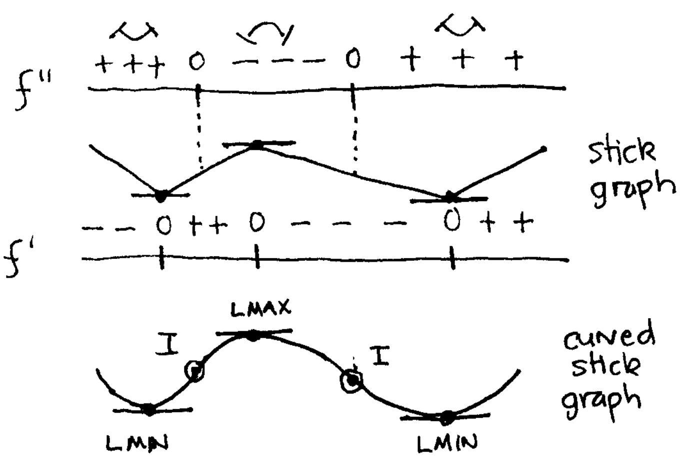
[This is a kink if $f' \to$ jump (finite)
zoom on graph looks like icons:]

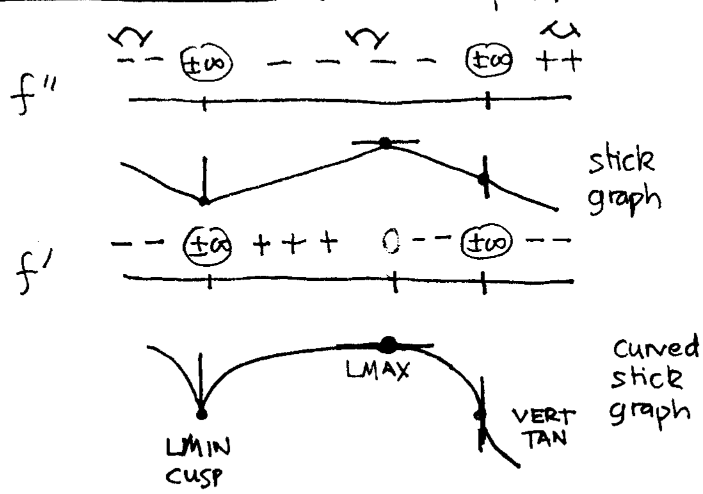
pts of inflection

(sign of f'' changes)

- $\neq$ hor, $\neq$ vert: typical
- hor: nice critical pts ($f' = 0$) which are not local extrema are pts of inf.
- vert: vertical tangent lines are pts of inf. (sign of f' doesnt change)

stick and curved stick graph from sign charts of f', f'' (two examples)





compute values of f at each key point, plot key points and any asymptotes, then "hang" curved stick graph on frame of key points/asymptotes to get caricature graph of f

road map building: two examples (domain: all real numbers)

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2)$

$= 12x(x+1)(x-2)$

$= 12(x-(-1))(x-0)(x-2)$

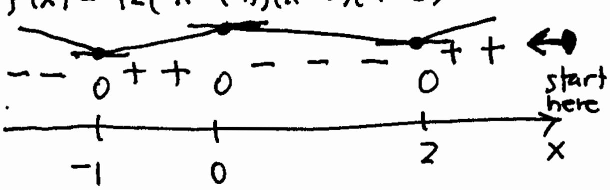
$f''(x) = 36x^2 - 24x - 24 = 12(3x^2 - 2x - 2)$

$= 36\,(x - x_-)(x - x_+)$

$x_\pm = \frac{2 \pm \sqrt{4+4\cdot 6}}{6} = \frac{1 \pm \sqrt{7}}{3} \approx -.55, 1.22$

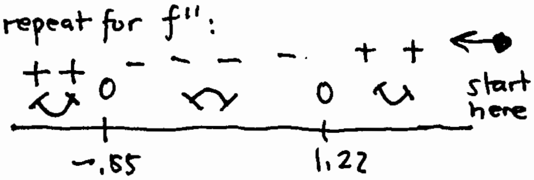
■ examine sign of f':

$f'(x) = 12(x-(-1))(x-0)(x-2)$



to determine sign by thinking work from right to left. to right of all zeros, all differences positive & leading coeff pos. so $f' > 0$. Crossing 2, factor $(x-2)$ changes sign. Crossing 0, factor $(x-0)$ changes sign. Crossing −1, factor $(x-(-1))$ changes sign. AND each factor only changes sign as you cross its zero & no signs can change in between the zeros.

■ repeat for f'':



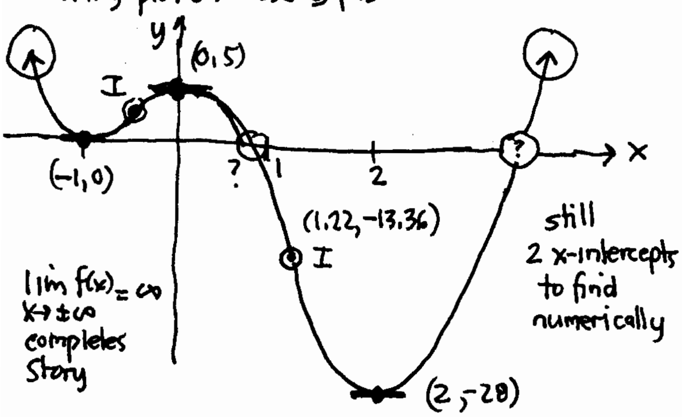
$f''(x) \approx 36(x-(-.55))(x-1.22)$

(2 pts of inflection where concavity changes)

SEE CURVED STICK FIGURE GRAPH ON PAGE 1.

■ evaluate f at 3 critical pts, 2 pts of inflection to get 5 pts to plot, plus y-intercept $(0,5)$ is a crit pt. already.

"Hang plot on these 5 pts"



$f(x) = x^{2/3}(6-x)^{1/3}$

x-intercepts $= 0 \to x = 0, 6$

$f'(x) = \cdots = \frac{4-x}{x^{1/3}(6-x)^{2/3}}$

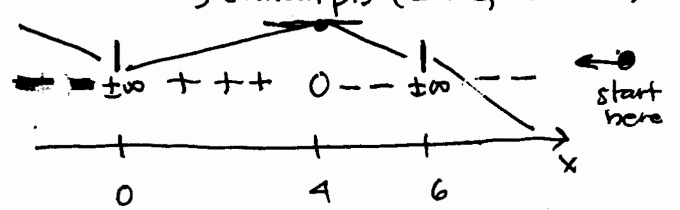
$f''(x) = \cdots = \frac{-8}{x^{4/3}(6-x)^{5/3}}$

■ examine sign of f': can change across zero or when undefined.

odd $\to 4-x \to\ = 0$ $(f' = 0) \to x = 4$

$\underbrace{x^{1/3}}_{\text{odd}}\underbrace{(6-x)^{2/3}}_{\text{even}} \to\ = 0$ $(|f'| \to \infty) \to x = 0, 6$

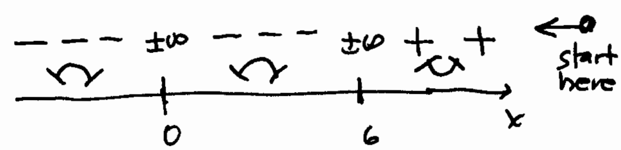
3 critical pts (1 nice, 2 not nice)



oops

$x > 6$ makes 1 factor negative, sign −. crossing $x=6$ no change in sign of even power. crossing $x=4$ changes sign of $(4-x)$, now + again. crossing $x=0$ changes sign (odd power). → so cusp at 0, vert tangent at 6, local max at 4.

■ repeat for f'':



$\frac{-8}{\underbrace{x^{4/3}}_{\text{even}}\underbrace{(6-x)^{5/3}}_{\text{odd}}}$

$x > 6$, odd power $(6-x)$ is negative, extra minus so sign is $+$.

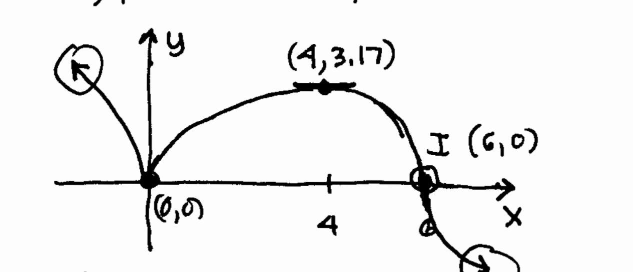
crossing $x=6$ changes sign to −.

crossing $x=0$ doesnt change sign.

one pt of inflection. SEE CURVED STICK FIGURE GRAPH

■ evaluate f at 3 critical pts, one of which is a pt of inflection, one an x-intercept

"hang plot on these 3 pts"



$\lim_{x\to-\infty} f(x) = \infty$ $\quad$ $\lim_{x\to+\infty} f(x) = -\infty$