Understanding Function Notation:

functions within expressions or with expressions within the function

opening and closing delimiters for argument

$f(x) = 2x^2 - 1$ ← formula defining rule of function

"enter x, square it, double result, subtract 1"

dummy variable standing for input ↔ $f(t) = 2t^2 - 1$ just as good

name of function

value of function on input x

unwritten parentheses are understood to surround the input variable in the function within its formula, and to surround the function expression itself if it occurs inside a larger formula:

$$(f((x))) = (2(x)^2 - 1)$$

[parentheses control the order of operations in mathematical expressions]

to make sure operations are applied correctly to arguments which are themselves expressions

to make sure operations are applied correctly to this expression when inside a larger expression

$$f(x) = 2(x)^2 - 1$$

$$f(x+1) = 2(x+1)^2 - 1$$ ← add to argument of function

add to value of function

$$f(x) + 1 = (2(x)^2 - 1) + 1$$

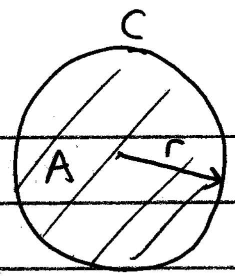
$$f(x+1) - f(x) = (2(x+1)^2 - 1) - (2(x)^2 - 1)$$

what is "inside" and "outside" of the original function delimiters cannot be combined by any rule of algebra:

$$\tfrac{1}{2} f(2x) \neq f(x)$$

$$f(x+1) - 1 \neq f(x).$$

FUNCTIONAL RELATIONSHIPS



On the circle the 3 variables r (radius), C (circumference), and A (area) are all related to each other — each can be expressed in terms of either of the other two variables — only one is "independent."

$$C = 2\pi r \rightarrow r = C/2\pi \longrightarrow A = \pi\left(C/2\pi\right)^2 = \frac{C^2}{4\pi}$$

$$A = \pi r^2 \rightarrow r = \sqrt{A/\pi} \longrightarrow C = \sqrt{(2\pi)}\sqrt{A/\pi} = 2\sqrt{\pi A}$$

$r \geq 0,\ C \geq 0,\ A \geq 0$
physical dimensions

$\boxed{\text{Summary}}$

$$A = \underbrace{\pi r^2}_{f(r)} = \underbrace{\frac{C^2}{4\pi}}_{h(C)}$$

$$C = \underbrace{2\pi r}_{g(r)} = \underbrace{2\sqrt{\pi A}}_{k(A)}$$

$$r = \underbrace{\frac{C}{2\pi}}_{j(C)} = \underbrace{\sqrt{\frac{A}{\pi}}}_{\ell(A)}$$

inverse function pairs

$f(x) = \pi x^2$
$g(x) = 2\pi x$
$h(x) = \frac{x^2}{4\pi}$
$j(x) = \frac{x}{2\pi}$
$k(x) = 2\sqrt{\pi x}$
$\ell(x) = \sqrt{\frac{x}{\pi}}$

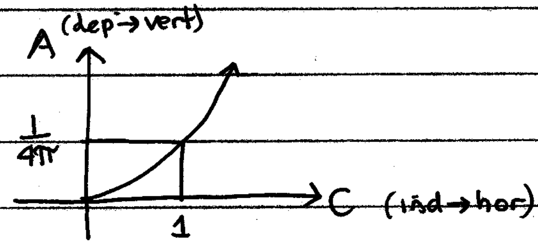
conclusions:

$A = f(r)$	$C = g(r)$	$A = h(C)$
$r = \ell(A)$	$r = j(C)$	$C = k(A)$
$\therefore$ f and ℓ are inverses	$\therefore$ g and j are inverses	$\therefore$ h and k are inverses

~~6 different functions~~ describe the relationships among these 3 variables.

parametrized curve interpretation (eliminate r)

ind. $C = 2\pi r \rightarrow r = C/2\pi$
dep. $A = \pi r^2 \longrightarrow A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$



or

ind $A = \pi r^2 \longrightarrow A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi} \rightarrow$ solve
dep $C = 2\pi r \rightarrow r = \frac{C}{2\pi}$

$C^2 = 4\pi A,\ C = \sqrt{4\pi A} = 2\sqrt{\pi A}$