1. The curve \( y = -h \ln \left( 1 - \frac{x}{R} \right) \) over the interval \( 0 \leq x < R \) is rotated around the \( x \)-axis, with \( h > 0, R > 0 \).

a) Let \( H > 0 \) designate the average value of \( y \) over this interval. Write down a simplified definite integral for \( H \) and then evaluate it by hand showing all your steps, and make sure it agrees with Maple's evaluation.

b) Make a diagram of this curve over this interval and include a horizontal line for this average value. Does the rectangle it makes appear to have the same area as that above the curve? Explain.

c) Let \( V \) denote the volume of a solid formed by revolving this curve segment around the \( x \)-axis, and indicate a typical vertical cross-section of the integration region needed to evaluate this volume, labeling its endpoints appropriately to justify your limits of integration and indicating how you obtained your integrand. Write down a simplified definite integral for \( V \) and then use Maple to evaluate it.

d) Compare this solid with a cylinder of revolution about the \( x \)-axis. What radius would the cylinder need to have for it to have the same volume?

\[ H = \frac{1}{R} \int_{0}^{R} -h \ln \left( 1 - \frac{x}{R} \right) \, dx \]

\[ \int \ln \left( 1 - \frac{x}{R} \right) \, dx = \int \ln u \, \frac{1}{u} \, du \]

\[ \frac{du}{dx} = 1 - \frac{x}{R}, \quad x = 1 - u \]

\[ = x \ln \left( 1 - \frac{x}{R} \right) - \int \frac{x \, dx}{R - x} \]

\[ = x \ln \left( 1 - \frac{x}{R} \right) - \left[ \frac{xR}{x} \ln x - \frac{xR}{x} \right] \]

\[ = (x - R) \ln \left( 1 - \frac{x}{R} \right) + R - x \]

\[ H = -h_0 \int_{0}^{R} \ln \left( 1 - \frac{x}{R} \right) \, dx = \left. \left[ x \ln (1 - x) - (-1)(x - 1) \right] \right|_{0}^{R} \]

\[ = h_0 \]

\[ V = \pi \int_{0}^{R} \left( h (1 - \frac{x}{R}) \right)^2 \, dx \]

\[ \Rightarrow \]

\[ \text{Maple} \]

\[ V = \frac{2h^2 \pi R^2}{2} \]

\[ r = \sqrt{2h} \]