Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use arrows and equal signs when appropriate. Always simplify expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (not decimal approximations, if possible).

1a) Evaluate \( \int x e^{-x^2/2} \, dx \) by hand.

b) Use the result of part (a) to evaluate \( I = \int_0^\infty x e^{-x^2} \, dx \) exactly (no decimal approximations).

c) Make a rough sketch of the region whose area is represented by this integral (telemetry).

d) Evaluate the trapezoidal approximation \( T_4 \) of this integral \( \left( n = 4 \text{ divisions} \right) \).

e) Evaluate \( I \) numerically (decimal equivalent) and the error \( E_4 = |I - T_4| \).

f) Compute the error bound \( |E_4| \leq \frac{K}{12n^2} \), where \( n = 4 \) and \( K \) is the maximum value of the second derivative of the integrand \( (x^2) \) of integration (you may use approximate tools here).

What is \( T_4 \) within this error bound?

9) What value of \( n \) is required to make the error less than \( \frac{1}{2} \times 10^{-4} \)?