Use logarithmic differentiation to find the derivative of the function (entirely in terms of $x$!): $y = \sqrt[4]{\frac{x+1}{x-1}}$. Combine all powers in your final expression.

A particle moves along the curve $xy = 10$ in the plane, where length units are taken to be "cm." At what rate is $y$ decreasing when $y = 5\text{cm}$ if $x$ is increasing at the constant rate of $\frac{1}{2} \text{ cm/s}$?

- a) Make a picture describing this situation and indicating the particle position o on the curve and its direction of motion $\rightarrow$ (and as usual: label your graph completely).
- b) Translate the question posed by this word problem (lite!) into derivative notation $\frac{dy}{dt} = ?$, where $C$ is the condition describing when/where the evaluation takes place.
- c) Calculate this quantity and consider answer the question posed in a complete English sentence.

$$\ln y = \ln \left(\frac{x^2 + 1}{x - 1}\right)^{\frac{1}{4}} = \frac{1}{4} \left[ \ln (x^2 + 1) - \ln (x - 1) \right]$$

expanding using log rules

$$\frac{1}{y} \frac{dy}{dx} \ln y = \frac{1}{4} \left[ \frac{1}{x^2 + 1} \cdot 2x - \frac{1}{x - 1} \cdot 1 \right]$$

constant factor / sum rules

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{4} \left[ \frac{2x}{x^2 + 1} - \frac{1}{x - 1} \right] = \frac{1}{4} \left[ \frac{(x - 1) - (x^2 + 1)}{x^2 + 1 \cdot (x - 1)} \right] = \frac{1}{4} \left[ \frac{2}{x^2 + 1 \cdot (x - 1)} \right]$$

log rule, chain rule

$$\frac{1}{4} \left[ \frac{2}{x^2 + 1 \cdot (x - 1)} \right]$$

factor combine fractions cancel

$$\frac{dy}{dx} = -\frac{x}{y} \left(\frac{x^2 + 1}{(x - 1)^{\frac{3}{4}}} \frac{1}{4}\right)$$

substitution exponential rule

$$\frac{dy}{dx} = -\frac{\text{end expression}}{(x^2 + 1)^{\frac{1}{4}} (x - 1)^{\frac{3}{4}}}$$

combine powers

\[ y \atop x \] is decreasing at $1.25\text{cm/s}$ when $y = 5\text{cm}$. 