A capacitor charges to its maximum charge \( Q_{\text{max}} = 1000 \) in a fraction of a second following the relation between charge \( Q \) in some units and time \( t \) in seconds.

\[ Q = 1000 \left( 1 - e^{-100t} \right) \]

a) Express \( t \) as a function of \( Q \).

b) Use your result to determine the time it takes the capacitor to reach 50% of its maximum charge. Give your final answer with units.

c) Repeat for 99%.

\[ Q = 1000 \left( 1 - e^{-100t} \right) \]

\[ \frac{Q}{1000} = 1 - e^{-100t} \]

\[ \ln \left( e^{-100t} \right) = \ln \left( 1 - \frac{Q}{1000} \right) \]

\[ -100t = \ln \left( 1 - \frac{Q}{1000} \right) \]

\[ t = -\frac{1}{100} \ln \left( 1 - \frac{Q}{1000} \right) \]

**Note:**

\[ 1 - \frac{Q}{1000} > 0 \]

\[ 1 > \frac{Q}{1000} \]

\[ 1000 > Q \quad \text{or} \quad Q < 1000 \]

In fact, since the relationship holds only for \( t \geq 0 \), then \( Q \geq 0 \), and the domain of this function is \( 0 \leq Q < 1000 \). Obvious from the range of allowed values for \( Q \) in the graph.

\[ Q = .50 \cdot 1000 \rightarrow \]

\[ t = -\frac{1}{100} \ln \left( 1 - \frac{.50}{1000} \right) = -\frac{1}{100} \ln \left( .50 \right) \approx \frac{0.69}{100} \text{sec} \]

(about 7 milliseconds)

\[ Q = .99 \cdot 1000 \rightarrow \]

\[ t = -\frac{1}{100} \ln \left( 1 - \frac{.99}{1000} \right) = -\frac{1}{100} \ln \left( .01 \right) \]

\[ = \frac{1}{100} \ln \left( 0.01^\frac{1}{100} \right) \approx \frac{1}{100} \ln 100 \approx 0.46 \text{sec} \]

(about 46 milliseconds)