Show all work, including mental steps, in a clearly organized way that speaks for itself.
Use proper mathematical notation, identifying expressions by their proper symbols
(introducing them if necessary) and using equal signs or arrows when appropriate.

1. \( f(x, y) = \sqrt{x^2 + y^2} = (x^2 + y^2)^{\frac{1}{2}} \)
   a) Find the linear approximation \( L(xy) \) to \( f \) at \( x = 5, \ y = 12 \).
   b) Use it to approximate the number \( \sqrt{(4.8)^2 + (12.2)^2} \).

2. The two legs of a right triangle are measured to be 5 m and 12 m with a possible error in measurement of at most 0.2 cm in each. Use differentials to estimate the error in the calculated value of the length of the hypotenuse.
   (Be sure to use consistent units: m or cm.)

   \( \frac{df}{dx} = \frac{x}{x^2 + y^2} \) 
   \( \frac{df}{dy} = \frac{y}{x^2 + y^2} \)

   a) \( f(5,12) = (5^2 + 12^2)^{\frac{1}{2}} = 13 \)
   \( f_x(5,12) = \frac{5}{(5^2 + 12^2)^{\frac{1}{2}}} = \frac{5}{13} \)
   \( f_y(5,12) = \frac{12}{(5^2 + 12^2)^{\frac{1}{2}}} = \frac{12}{13} \)

   \[ L(xy) = f(5,12) + f_x(5,12)(x-5) + f_y(5,12)(y-12) \]
   \[ = 13 + \frac{5}{13}(x-5) + \frac{12}{13}(y-12) \]

   \( \sqrt{(4.8)^2 + (12.2)^2} = f(4.8,12.2) \approx L(4.8,12.2) = 13 + \frac{5}{13}(4.8-5) + \frac{12}{13}(12.2-12) \)
   \[ = 13 + 0.2 \frac{-0.2}{13} = 13 + 0.015 \approx 13.1 \)

   b) \( dh = \frac{dx}{dx} \frac{df}{dx} + \frac{dy}{dy} \frac{df}{dy} \)
   \( = \frac{x dx + y dy}{\sqrt{x^2 + y^2}} \)

   \( dh \bigg|_{x=5} \ y=12 = \frac{5 dx + 12 dy}{13} \)

   \( dh \bigg|_{x=5} \ y=12 = \frac{5(0.002) + 12(0.002)}{13} = (0.002)(17) \approx 0.026 \)

   Note: If you forget about units and use \( x=5, y=12, \ dx=0.2, \ dy=0.2 \) and get \( dh = 0.26 \), it is clear that the units of \( dh \) are cm in agreement with \( dx \) and \( dy \). This is what I did myself initially.