Find the center \( C(x_0, y_0, z_0) \) and radius \( r \) of the sphere described by the equation \( x^2 + y^2 + z^2 + 2x + 8y - 4z = 28 \).

a) Represent the position vector \( \vec{r}_0 = <x_0, y_0, z_0> \) of the center you found with respect to a set of Cartesian axes, as in this diagram: (label tick-marks on the axis), and indicate the angle \( \theta \) between \( \vec{r}_0 \) and the positive \( z \)-axis.

c) Make a wild guess from your diagram about the value of \( \theta \) in degrees.

d) Find an expression for the exact angle \( \theta \) in radians.

e) Give a decimal approximation to your angle in degrees, keeping only one decimal place.

\[ (x+1)^2 - 1 + y^2 + 8y + (z-2)^2 - 2^2 = 28 + 1 + 16 + 4 = 49 = 7^2 \]

\[ \vec{r}_0 = <-1, -4, 2> \]

\[ \cos \theta = \frac{\vec{e} \cdot \vec{r}_0}{|\vec{e}| |\vec{r}_0|} = \frac{0 \cdot 0 + 1 \cdot (-4) + 2 \cdot 2}{\sqrt{21}} \]

\[ \theta = \arccos \left( \frac{2}{\sqrt{21}} \right) \approx 64.1^\circ \]

While high school trig shows that \( \theta = \arctan \left( \frac{\sqrt{17}}{2} \right) \), remember we are trying to acquire familiarity with dot product geometry here. If I had instead asked for the angle between the position vectors of the center and the sphere north pole \( <-1, -4, 2+7> \) only the dot product approach would make sense. Try to use generic techniques on quizzes & tests, not tricks to get the answer.

\[ \text{evalf} \left( \arccos \left( \frac{2}{\sqrt{21}} \right) \times 180/\pi \right) \text{ (or NYSIWYG entry, right-click, approximate)} \]