1. \( f(x) = 3x^4 - 4x^3 - 12x^2 + 5 \) Taylor expand about \( x = 1 \) and then about \( x = 2 \).

Check with MAPLE and print out your verification.

2. Stewart II, R.58: The force due to gravity on an object with mass \( m \) at a height \( h \) above the surface of the Earth is

\[ F = \frac{mgR^2}{(R+h)^2}, \]

where \( R \) is the radius of Earth and \( g \) is the acceleration due to gravity.

a) Express \( F \) as a series in powers of \( h/R \).

b) Observe that if we approximate \( F \) by the first term in the series, we get the expression \( F \approx mg \) that is usually used when \( h \) is much smaller than \( R \). Use the Alternating Series Estimation Theorem to estimate the range of values of \( h \) for which the approximation \( F \approx mg \) is accurate to within 1%. (Use \( R = 6400 \text{ km} \).)

Hint: Use \( (1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots \) with \( x = h/R \). First Taylor expand \( f(x) = (1+x)^{-2} \) and easily get formula for \( n \)th term.

\[
\begin{align*}
\int f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(0)(x-1)^n}{n!} \\
&= \int \left[ -8 - 24(x-1) - 12(x-1)^2 + \frac{48}{3} (x-1)^3 + \frac{72}{4} (x-1)^4 \right] \\
&= \int \left[ -8 - 24(x-1) - 6(x-1)^2 + 8(x-1)^3 + 3(x-1)^4 \right] \\
&= f(x) - 8 \cdot 24 - 6 \cdot 24 + 8 \cdot 24 + 3 \cdot 24 - 27 + 72(x-2)^2 + 48(x-2)^3 + 12(x-2)^4 \\
&= f(x) + 27 + 36(x-2)^2 + 20(x-2)^3 + 3(x-2)^4 \\
&= \text{Taylor}(f(x), x=1, 5); \\
&= \text{Taylor}(f(x), x=2, 5);
\end{align*}
\]

2. \( F = \frac{mgR^2}{(R+h)^2} = \frac{mgR^2}{R^2(1+\frac{h}{R})^2} = mg(1+\frac{h}{R})^{-2} = mg(1+x)^{-2} \)

\[
\begin{align*}
f(x) &= (1+x)^{-2} \\
f^{(0)}(x) &= 1 \\
f^{(1)}(x) &= -2x \\
f^{(2)}(x) &= -2 \\
f^{(3)}(x) &= 6x \\
f^{(4)}(x) &= +12 \\
f^{(5)}(x) &= -24 \\
f^{(6)}(x) &= +36 \\
f^{(7)}(x) &= -48 \\
f^{(8)}(x) &= +72 \\
f^{(9)}(x) &= -84 \\
f^{(10)}(x) &= +96 \\
f^{(11)}(x) &= -108 \\
&= \sum_{n=0}^{\infty} (-1)^n n! x^n \\
&= mg \sum_{n=0}^{\infty} (-1)^n (n+1)! (\frac{h}{R})^n \\
&= mg \left( 1 - 2(\frac{h}{R}) + 3(\frac{h}{R})^2 - \cdots \right) \\
&= mg \left( 1 - 2(\frac{h}{R}) + (\frac{h}{R})^2 - \cdots \right) \\
&= mg - \frac{2mh}{R} + \cdots
\end{align*}
\]

\[
\begin{align*}
2\frac{mh}{R} &\leq 0.01 mg \quad (\text{error less than } 1\% \text{ of } mg) \\
2\frac{h}{R} &\leq 0.01 R = 0.01(6400 \text{ km}) = 32 \text{ km}
\end{align*}
\]

\( h \leq 32 \text{ km} \)