Show all work on this sheet, including indications of mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation/syntax. Label parts, box find short answers.

0) \( S = \sum_{n=1}^{\infty} \frac{1}{n^{11}} \) (eleventh power)

a) Justify the convergence of this infinite series.

b) Write out the 5th partial sum exactly and give its decimal equivalent.

c) Estimate the truncation error \( R_5 \) using the integral remainder approach.

d) How many terms do you need to get 10 decimal place accuracy? Show all work.

0) On the basis of a), evaluate \( S \) to 10 decimal place accuracy.

0a) This is a p-series \( \sum_{n=1}^{\infty} \frac{1}{n^p} \) with \( p = 11 > 1 \) and so converges.

\[
\text{[since the corresponding integral converges: } \int_{1}^{\infty} x^{-11} \, dx = \frac{x^{-10}}{-10} \bigg|_1^\infty = \frac{1}{10} \]\]

b) \( S_5 = 1 + \frac{1}{2^{11}} + \frac{1}{3^{11}} + \frac{1}{4^{11}} + \frac{1}{5^{11}} \approx 1.000494184 \)

c) \( R_5 < S - S_5 = \int_{5}^{\infty} x^{-11} \, dx = \frac{x^{-10}}{-10} \bigg|_5^\infty = -\frac{1}{10 \cdot 5^{10}} + \frac{1}{10 \cdot 5^{10}} = 0 \)

\[
\approx 1.024 \times 10^{-7} \]

d) \( R_n < S - S_n = \int_{n}^{\infty} x^{-11} \, dx = \frac{x^{-10}}{-10} \bigg|_n^\infty = \frac{1}{10 \cdot n^{10}} < \frac{1}{2} \times 10^{-10} \)

Take reciprocals: \( 10 \cdot n^{10} > 2 \cdot 10^{10} \)

\( n^{10} > \frac{2}{10} \cdot 10^{10} \)

\( n > \left( \frac{2}{10} \cdot 10^{10} \right)^{\frac{1}{10}} = \left( \frac{2}{10} \right)^{\frac{1}{10}} \cdot 10 = \left( \frac{1}{5} \right)^{\frac{1}{10}} \cdot 10 \)

\( \approx 8.513 \)

So \( n = 9 \) is the least number of terms.

e) \( S_9 = \sum_{n=1}^{9} \frac{1}{n^{11}} \approx 1.000494188 \)