\( \theta(t) = At e^{-ct} \)

- **a)** Evaluate \( \int_0^t \theta(t) \, dt \)

- **b)** Show that \( A = c^2 \) makes \( \int_0^\infty \theta(t) \, dt = 1 \).

- **c)** Use calculus to find the time \( t_0 \) at which the probability density function peaks.

- **d)** What is the probability \( P(0 \leq t \leq t_0) \), i.e., evaluate \( \int_0^{t_0} \theta(t) \, dt \) for \( \theta(t) = c^2 t e^{-ct} \). Give the exact value and numerical approximation (floating point value to 3 decimal places) to this question.

- **e)** Repeat for \( P(0 \leq t \leq 2t_0) \).

- **f)** The value of \( t \) for which this cumulative probability is \( \frac{1}{2} \) is called the median: \( P(0 \leq t \leq t_m) = \frac{1}{2} \). From parts d) and e), does the median satisfy \( 0 \leq t_m \leq t_0 \) or \( t_0 \leq t_m \leq 2t_0 \) or \( 2t_0 \leq t_m \)?

- **g)** \( \frac{1}{2} = \int_0^{t_m} c^2 t e^{-ct} \, dt \)

Let \( U = ct \) and \( \overline{t_m} = c t_m \). Re-express this integral in terms of \( U \) and \( \overline{t_m} \). Notice that the result is equivalent to setting \( c = 1 \).

- **h)** For \( c = 1 \), plot \( \frac{1}{2} \) and the function of \( t_m \) which results from evaluating \( \int_0^{t_m} c t e^{-t} \, dt \) and determine the value of \( t_m \) at which the two curves intersect. [This is a check on f) since it tells you in units of to what the median is.]