Show all work on this sheet, including indications of your mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation/syntax.

Label parts if you box final short answers.

1) a) Evaluate \( \int x \sin 4x \, dx \)

b) Use part a) to evaluate \( \int_0^{\pi/4} x \sin 4x \, dx \).

c) Let \( f(x) = x \sin 4x \). Write down an expression \( S_4 \)

in terms of the Simpson rule \( n = 4 \) approximation to this definite integral.

If you forget the coefficients, give the \( n = 4 \) midpoint approximation instead.

d) Evaluate your expression from part c) using either MAPLE or your graphing calculator. Does your numerical result agree with part b)?

1) a) \[
\int x \sin 4x \, dx = \frac{x}{4} \cos 4x - \frac{1}{4} \int \cos 4x \, dx
\]

\[
= \frac{x}{4} \cos 4x + \frac{1}{16} \sin 4x + C
\]

b) \[
\int_0^{\pi/4} x \sin 4x \, dx = \left[ \frac{x}{4} \cos 4x + \frac{1}{16} \sin 4x \right]_0^{\pi/4}
\]

\[
= \frac{\pi}{16} \cos \pi - \frac{1}{4} \cos 0 + \frac{1}{16} \sin 0 = \frac{\pi}{16}
\]

c) \[
\Delta x = \frac{\pi/4}{4} = \frac{\pi}{16} = h
\]

\[
S_4 = \frac{1}{3} \left( \frac{\pi}{16} \right) \left( f(0) + 4f(\pi/16) + 2f(\pi/8) + 4f(\pi/4) + f(\pi/2) \right)
\]

\[> f(x) = x \rightarrow x \sin(4x) \]

\[> \text{evalf} \left( \left( \frac{\pi}{16} \right) \left( f(0) + 4f(\pi/16) + 2f(\pi/8) + 4f(\pi/4) + f(\pi/2) \right) \right) \]

\[> \text{evalf} \left( \frac{\pi}{16} \right) \]

\[> 0.1967971937 \]

[\[\text{error in 4th decimal place, pretty good. \[\text{Yes, results agree.} \]

\[\text{Note:} \]

\[
\int \cos ax \, dx = \frac{1}{a} \sin ax + C
\]

\[
\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C
\]

Coefficient of function input variable multiplies derivative (chain rule)

Divides antiderivative (variable substitution)