Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation/syntax. Identify expressions with their proper symbols (introduce them if necessary). Label parts. **Box** final short answers requested.

a) Evaluate \( I = \int_0^1 \int_{2x}^3 x^2 + y \ dy \ dx \)

\[
I = \int_0^1 \int_{2x}^3 (x^2 + y) \ dy \ dx
\]

\[
= \left[ xy + \frac{y^2}{2} \right]_{y=2x}^{y=3}
= x^2(3) + \frac{3^2}{2} - x^2(2x) - \frac{(2x)^2}{2}
= 3x^2 - 3x^3 + \frac{9}{2} - 2x^2
\]

\[
= \int_0^1 \left( \frac{9}{2} - \frac{3}{2} x^2 - 3x^3 \right) dx
= \left[ \frac{9}{2} x - \frac{3}{2} \frac{x^3}{3} - \frac{3x^4}{4} \right]_0^1
= \frac{9}{2} - \frac{1}{2} - \frac{3}{4} = \frac{8 - 3}{4} = \frac{5}{4}
\]

b) Make a diagram illustrating this iterated integral:

shade region of integration, label endpoints of cross-section line segment indicating inner integral, endpoints of outer integral.

Now redo the diagram to represent the reversed order of integration.

Re-evaluate the new integral.

Do parts a) and d) agree as they should? (Maple will tell you the correct answer).

c) \( I = \int_0^3 \int_0^{y/3} x^2 + y \ dx \ dy \)

\[
I = \int_0^3 \int_0^{y/3} x^2 + y \ dx \ dy
\]

\[
= \left[ \frac{x^3}{3} + y \right]_{x=0}^{x=y/3}
= \left( \frac{y^3}{3} \right)^3 + y \left( \frac{y}{3} \right) - 0
\]

\[
= \frac{y^3}{3} + \frac{y^2}{3}
\]

\[
= \int_0^3 \left( \frac{y^3}{3} + \frac{y^2}{3} \right) \ dy
= \left[ \frac{y^4}{4} \right]_0^3
= \frac{3^4}{4} + \frac{3^3}{3^2} = \frac{81}{4} + \frac{27}{3} = \frac{13}{4}
\]