1. \( f(x, y) = 5xy^2 - 4x^3y \)
   
   a) Find the directional derivative of \( f \) in the direction of the vector \( \mathbf{v} = \langle 4, 3 \rangle \) at the point \( (1, 2) \).
   
   b) Find the maximum rate of change of \( f \) at the point \( (1, 2) \) and a unit vector \( \mathbf{U} \) giving the direction in which this maximum occurs.
   
   c) Write an equation for the tangent plane to the graph \( z = f(x, y) \) at the point \( (1, 2) \). Give a simple normal vector \( \mathbf{n} \) to this plane.

2. The voltage \( V \) in a simple electrical circuit is slowly decreasing as the battery wears out. The resistance \( R \) is slowly increasing as the resistor heats up. Use Ohm's law \( I = V/R \) to find how the current \( I \) (measured in amperes; A) at the moment when \( R = 400 \Omega, V = 32 \text{ volts} \), \( dV/dt = -0.01 \text{ volts/s} \) and \( dR/dt = 0.03 \Omega/s \). Specify units in your answer.

3. Find three positive numbers \( x, y, z \) whose sum is 30 and whose product \( P \) is a maximum. Be sure to confirm your claim with the second derivative test.

4. Find the linear approximation \( L(x, y, z) \) to the function
   
   \( f(x, y, z) = x^3y^2 + z^2 \) at the point \( (2, 3, 4) \) and use it to estimate the number \( (1.98)^3 \sqrt{(3.01)^2 + (3.97)^2} \).