MAT1500-03/11 Quiz 4  Print Name (Last, First) 

Show all work on this sheet, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation. [Box] final short answers.

\[ f(x) = 2x^2 - 3 \quad , \quad g(x) = \frac{3x^2 - 1}{1 - x^2} \]

1) a) Find the slope of the tangent line to \( f \) at \( a \) (or at \( "x=a" \)). [using limits]
   
   b) Use your result to write the equation of the tangent line to \( f \) at \( x=1 \), and give your final result with \( y \) expressed as a function of \( x \).

2) a) Evaluate the limits necessary to determine if \( g \) has any horizontal asymptotes.
   
   b) Give the equations of any horizontal asymptotes you find.

0) a)  
   \[ m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \]
   
   \[ = \lim_{h \to 0} \frac{(2(a+h)^2-3) - [2a^2-3]}{h} \]
   
   \[ = \lim_{h \to 0} \frac{2a^2 + 4ah + 2h^2 - 3 - 2a^2 + 3}{h} \]
   
   \[ = \lim_{h \to 0} \frac{4ah + 2h^2}{h} \]
   
   \[ = 4a + 2h \]
   
   so \( m = \lim_{h \to 0} (4a + 2h) = 4a + 2(0) = 4a \)

b) Setting \( a = 1 \):
   
   \( m = 4(1) = 4 \).
   
   If \( x = 1 \), then \( y = f(1) = 2(1)^2 - 3 = 2 - 3 = -1 \)
   
   Pt \((1, -1)\), slope 4 \( \xrightarrow{\text{pt-slope eq:}} \)
   
   \[ y - (-1) = 4(x - 1) \]
   
   \[ y + 1 = 4x - 4 \]
   
   \[ y = 4x - 5 \]

2) a)  
   \[ \lim_{x \to \infty} \frac{3x^2 - 1}{1 - x^2} = \lim_{x \to \infty} \frac{3x^2 - 1}{1 - x^2} \times \frac{x^2}{x^2} = \lim_{x \to \infty} \frac{3 - \frac{\text{y}}{x^2}}{1 - \frac{\text{y}}{x^2}} = \frac{3}{-1} = -3 \]
   
   \[ \lim_{x \to -\infty} \frac{3x^2 - 1}{1 - x^2} = -3 \] (same calculation, no change since \( g \) is even)

b) \( y = -3 \) is a horizontal asymptote for \( g \) both as \( x \to \infty \) and \( x \to -\infty \).