A 30 year old woman accepts an engineering position with a starting salary of $30,000 per year. Her salary, S(t), increases exponentially with \( S(t) = 30e^{0.05t} \) thousand dollars after \( t \) years.

Meanwhile, 12% of her salary is deposited continuously in a retirement account, which accumulates interest at a continuous annual rate of 6%.

1. Estimate \( A \) in terms of \( S \) to derive the DEQ satisfied by the amount \( A(t) \) in her retirement account after \( t \) years.
2. Compute \( A(40) \), the amount available for her retirement at age 70.

Think of \( dt \) as a paycheck period (typically 2 weeks \( \approx \frac{1}{52} \text{ yr} \))

\[
\frac{dA}{dt} = 0.12 \left( \frac{500}{\text{yr}} \right) \frac{dt}{\text{yr}} + 0.06 A(t) \frac{dt}{\text{yr}}
\]

Similarly, defining \( A(t) \) as retirement fund deposit that paycheck

we have (continuous depositing is obviously idealization, but it gives us a simple model! (interest rates are rarely constant etc, so none of the numbers can be trusted in the real world.)

however, it allows us to take the limit \( dt \to 0 \) to get a DEQ rather than just some discrete equation:

\[
\frac{dA}{dt} = 0.12 \left( 30e^{0.05t} \right) + 0.06 A(t)
\]

\[
\text{Linear DEQ:} \quad \frac{dA}{dt} = 1200e^{0.05t} + 0.06 A(t)
\]

\[
\text{Solve:} \quad A(t) = 1200 \left( e^{0.05t} - 1 \right) + C e^{-0.06t}
\]

Find \( A(0) = 0 \) and \( C = \frac{1200}{e^{0.05} - 1} \)

\[
A(t) = 1200 \left( e^{0.05t} - 1 \right) + \frac{1200}{e^{0.05} - 1} e^{-0.06t}
\]

Understandable approach to retirement account

How to factor in inflation? Real dollar value?