characteristic constants for exponential behavior

\[ y = \sin x \]

periodic; each "cycle" repeats every interval of length \(2\pi\)

\[ \frac{Q}{Q_0} \]

Q = \(Q_0 \sin kt\) or \(Q_0 = \sin kt\) if time

\[ \frac{Q_0}{Q} = \sin kt \]

"frequency" \(k > 0\)

rate parameter

units: inverse time

\[ \sin \left(\frac{kt}{\pi}\right) = 2\pi \text{ at end of cycle} \]

\[ t = \frac{2\pi}{k} \]

period \(T\)

sets scale for "window";

cycle in horizontal width (time interval)
tells how many cycles you see

Similarly:

\[ y = e^{-x} \]

\[ y = e^x \]

\[ Q = Q_0 e^{kt} \]

\[ Q = Q_0 e^{kt} \]

\(k\) is the rate factor (units: inverse time)

\[ Q = Q_0 e^{kt} \text{ or } \frac{Q}{Q_0} = e^{kt} \]

now length (or time) scale can be characterized by how long it takes to increase/decrease by a factor of \(e\) or \(2\) (two special numbers):

\[ e = e^1 \]

\[ e^{kt} \]

\(k > 0\)

\(k < 0\)

\[ e^{kt} = e \rightarrow kt = 1 \rightarrow t = \frac{1}{k} \]

\[ e^{-kt} = e \rightarrow kt = -1 \rightarrow t = -\frac{1}{k} = \frac{1}{|k|} \]

\[ e = e^0 \]

\[ e^{kt} \]

\(k > 0\)

\(k < 0\)

\[ e^{kt} = 2 \rightarrow kt = \ln 2 \rightarrow t = \frac{\ln 2}{k} \]

\[ e^{kt} = 2^{-1} \rightarrow kt = \ln 2 \rightarrow t = -\frac{\ln 2}{k} = \frac{(\ln 2)}{|k|} \]

\[ e = e^0 \]

\[ e^{kt} \]

\(k > 0\)

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Here what repeats every interval of length \(T\) (or \(\tau\))
is the doubling \((k > 0)\) / halving \((k < 0)\) of the value of \(Q\)
or the increase / decrease by a factor of \(e\).

no need to remember formulas for characteristic times - can just "rederive" them when working a problem.