Distance between a point and a line

Preliminary:

scalar component of \( \vec{b} \) perpendicular to \( \vec{a} \) is just
\[
|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta = |\vec{a} \times \vec{b}|
\]

compare with scalar component of \( \vec{b} \) along \( \vec{a} \):

Given a point \( P_1 \)
find any point \( P_2 \) on line (see next page)
evaluate difference vector \( \vec{P}_1 \vec{P}_2 \)
find its component perpendicular to the direction \( \vec{a} \) of the line:
\[
D = |\vec{a} \times (\vec{P}_1 \vec{P}_2)|
\]

Distance between a point and a plane

Given a point \( P_1 \)
find any point \( P_2 \) on the plane (next page)
evaluate the difference vector \( \vec{P}_1 \vec{P}_2 \)
find its scalar component along the direction \( \vec{n} \) normal to the plane
absolute value is the distance:
\[
D = |\vec{n} \cdot (\vec{P}_1 \vec{P}_2)|
\]

with these 2 projection operations we can extend these distance calculations easily to the distance between 2 parallel planes or between 2 skew lines (equivalent to 2 parallel planes) or between 2 parallel lines

(see next page)

we are not terribly interested in computing these distances. it only serves as practice with dot & cross products in projection operations
2. Distances between lines and planes

- 2 parallel planes: \( \vec{n}_1 \times \vec{n}_2 = 0 \)

\[
\begin{align*}
\vec{n}_1 & \quad \vec{n}_2 \\
\vec{r}_1 & \quad \vec{r}_2 \\
\vec{p}_1 & \quad \vec{p}_2 \\
\end{align*}
\]

\[
D = \left| \hat{n} \cdot (\vec{r}_1 - \vec{r}_2) \right|
\]

Component along \( \vec{n} \)

- 2 skew lines: \( \vec{a}, \vec{b} \) not proportional: \( \vec{a} \times \vec{b} \neq 0 \)

\[
\begin{align*}
\vec{a} & \quad \vec{b} \\
L_1 & \quad L_2 \\
\end{align*}
\]

Introduce planes

- 2 parallel lines: \( \vec{a}, \vec{b} \) proportional: \( \vec{a} \times \vec{b} = 0 \)

\[
\begin{align*}
\vec{a} & \quad \vec{b} \\
L_1 & \quad L_2 \\
\vec{r}_1 & \quad \vec{r}_2 \\
\end{align*}
\]

\[
D = \left| \hat{a} \times (\vec{r}_1 - \vec{r}_2) \right|
\]

Component perpendicular to \( \vec{a} \) or \( \vec{b} \)

One only needs to have a point on each plane or line to get a difference vector and then project that difference either along or perpendicular to the direction vector characterizing the plane or line to get a signed distance.

Point on a line:

\[
\begin{align*}
x &= x_0 + at \\
y &= y_0 + bt \\
z &= z_0 + ct
\end{align*}
\]

Point on a plane:

\[
a x + b y + c z = d
\]

All planes intersect at least one axis if \( c \neq 0 \) set \( x = y = 0 \) (\( z \)-axis) and solve for \( z \).