If a surveyor measures differences in elevation when making plans for a highway across a flat desert, corrections must be made for the curvature of the Earth. Let $R$ be the mean radius of the Earth (at sea level) and let $L$ be the length of the highway, assumed to be a circular arc at the surface of the Earth. $R = 6371$ km (Google).

a) Show that the correction $C$ is given by $C = R \left( \sec \left( \frac{R}{L} \right) - 1 \right)$.

[b] Hint: $C + R$ is the hypotenuse!]

b) By polynomial long division, find the first 3 nonzero terms in the Taylor series for \( \sec(x) = \frac{1}{\cos(x)} = \frac{1}{1 - \frac{x^2}{2} + \frac{x^4}{24} + \cdots} \). [Check with Maple’s taylor command.]

c) Then evaluate the first 2 nonzero terms in the Taylor series for \( \sec(x) - 1 \).

d) Now letting $x = L/R$ in this result, show that $C = R \left( \frac{L^2}{2R} + \frac{5L^4}{24R^4} \right)$.

e) If the highway were approximately at sea level we can use $R = 6371$ km.

For $L = 100$ km, evaluate $x = L/R$ and evaluate the two contributions $C_1 = C_1$ to the correction separately, giving the first term $C_1$ in meters and the second term $C_2$ in centimeters. How do these compare to a direct evaluation of part a) with technology?

f) Suppose we try to apply this to the 40-mile straightaway stretch of I-80 crossing the Bonneville Salt Flats east of Salt Lake City, Utah: elevation 4218 ft (from Google). Convert this added elevation to meters and add it to $R$ to obtain the new value appropriate to this scenario. Then using $L = 40$ mi (how many km?), recalculate the new values of $x = R/L$ and $C_1$ and $C_2$, again in meters and centimeters respectively.

Do we really need to worry about the elevation in this problem? Why?

Did your value for $C$ surprise you?