**TRIG**

Unit circle:
\[ x^2 + y^2 = 1 \]

\[ \cos^2 \theta + \sin^2 \theta = 1 \]

\[ (x, y) = (\cos \theta, \sin \theta) \]

**Reference Triangle**

- Vertical leg
- Horizontal leg
- Remember \( \theta \) increases in counterclockwise direction

\[ \sec \theta = \frac{1}{\cos \theta} \]
\[ \csc \theta = \frac{1}{\sin \theta} \]
\[ \tan \theta = \frac{\sin \theta}{\cos \theta} \]
\[ \cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta} \]

For each angle \( \theta \), we can draw a reference triangle hanging off the x-axis with acute reference angle \( \theta_0 \in [0, \frac{\pi}{2}] \).

The sign of the trig functions in the II, III, IV quadrants can be computed using the reference triangle.

**Example** Suppose \( \theta \) is in the II quadrant, i.e., \( \theta = \pi - \theta_0 \).

Then
\[ \sin \theta = y = \sin \theta_0 \]
\[ \cos \theta = -x = -\cos \theta_0 \]

The same argument can be used to get sign trig identities:

- Pretend \( \theta \) is \( 30^\circ \) to draw picture.
- Locate \( \pi - \theta \).
- Then from diagram
  \[ \sin(\pi - \theta) = \sin \theta \]
  \[ \cos(\pi - \theta) = -\cos \theta \]

  OR for \( -\theta \):
  \[ \sin(-\theta) = -\sin \theta \quad \text{(odd)} \]
  \[ \cos(-\theta) = \cos \theta \quad \text{(even)} \]

(Try it for \( \pi + \theta \)).

For example, pretend \( \theta \) is \( 30^\circ \):
\[ \cos(\frac{\pi}{2} - \theta) = \sin \frac{\pi}{2} \]
\[ \sin(\frac{\pi}{2} - \theta) = \cos \frac{\pi}{2} \]

(Try for \( \pi + \theta \)).
TRIG: SPECIAL ANGLES

"STANDING UP"

"LYING DOWN"

(Note $\frac{\sqrt{3}}{2} = \sqrt{1 - \left(\frac{1}{2}\right)^2}$ pythag. theorem )

These reference triangles give you the trig functions of $30^\circ$, $45^\circ$, $60^\circ$
or $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$.

(Recall $\frac{\theta_{\text{rad}}}{\pi} = \frac{\theta_{\text{deg}}}{180}$ or $\theta_{\text{rad}} = \frac{\pi}{180} \theta_{\text{deg}}$)

Putting reference triangles in all 4 quadrants gives us (together with the axes) 16 different angles all of whose trig functions can be calculated from the special angles we know.

\[ \cos \frac{5\pi}{6} = \cos \left(\pi - \frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2} \]

Angles in argument of trig functions in calculus are ALWAYS radians unless explicitly indicated to be degrees. As in $\cos 30^\circ \neq \cos 30$.

Degrees are just a tradition and have no mathematical origin—we use them only for convenience to visualize angles.

On a nonunit circle, length quantities scale by the radius $r$:
\[
\begin{align*}
  x &= r \cos \theta \\
  y &= r \sin \theta \\
  r &= \sqrt{x^2 + y^2} \\
  \tan \theta &= \frac{y}{x}
\end{align*}
\]

Relations between Cartesian & polar coordinates.