the only base you really need: \( e \)

1) exponential functions

\[ y = a^x = (e^{\ln a})^x = e^{(\ln a)x} \]

always convert to the natural exponential function with the identity \( a = e^{\ln a} \).

2) logarithmic functions

\[ y = \log_a x \quad \text{means} \quad x = a^y \]

\[ \ln x = \ln a^y \]

\[ = y \ln a \]

\[ y = \frac{\ln x}{\ln a} \quad \rightarrow \quad \log_a x = \frac{\ln x}{\ln a} \]

"\( y \) is the exponent you need to raise \( a \) to to get \( x \)."

use this identity to convert to natural log.

3) base 2: doubling times, half-life

population growth or radioactive decay can be described with either base 2 or base \( e \)

same growth curve, different growth rates with respect to base 2 and base 3

doubling time: \( \frac{1}{h} \)

characteristic time: \( \frac{1}{k} \)

\[ \ln \left( 2^{ht} = e^{kt} \right) \]

\[ \ln(2^{ht}) = \ln(e^{kt}) \]

\[ ht \ln(2) = kt \]

\[ k = (\ln 2) \cdot h \approx 0.693 \cdot h \]

\[ \frac{1}{k} = \left( \frac{1}{\ln 2} \right) \cdot \frac{1}{h} \approx 1.44 \cdot \frac{1}{h} \]

takes longer to grow by factor of \( e^{\approx 2.72} \) than a factor of 2.