Robert T. Jantzen: Research Narrative for Nonexperts

1978–2010

1 Formation

For a bright kid from a small town with a limited educational environment, my working class parents' sacrifice to pay high tuition to send me to a large nearby public high school started me on the road to an academic career, which almost by chance propelled me into Princeton University as an entering math major who quickly slid into theoretical physics while continuing to take many advanced mathematics courses. Having already been bitten by the relativity bug in ninth grade from a chance encounter with a playful book by Lillian and Hugh Lieber on Einstein's general theory of relativity (GR) which was a bit incomprehensible for me at the time, another chance event caught me up in the golden age of relativity at Princeton. This was after all Einstein's home for the final decades of his life, and his friendship with John Wheeler (an iconic Princeton physicist and my sophomore modern physics teacher) led to the latter's revival of interest in relativity there, allowing me to begin working as an undergraduate on mathematical cosmology, a subject born at Princeton decades earlier first by Einstein's friend Kurt Gödel whom I was privileged to consult with one afternoon at the Institute for Advanced Study (IAS), and shortly after expanded by Abe Taub who had been a joint math/physics doctoral student in relativity when Einstein first arrived at the IAS, later followed by Gödel. Remo Ruffini, an Italian colleague of Wheeler working on the newly named 'black holes,' invited me to work on a special project translating a 90 page paper in Italian by Luigi Bianchi (1898) that formed the mathematical foundation of the later work of Gödel and Taub in their application of the geometry of homogeneous Riemannian 3-spaces to spatially homogeneous cosmology. Thus began my GR career and lifetime love affair with Italy.

This area of relativity physics explores mathematical models of the universe within Einstein's general theory of relativity in which the spaces at each given time are homogeneous but in general anisotropic, namely all points of space at a given time are equivalent, but at a given point all directions are not equivalent in general. This equivalence of points in space is characterized by a "Lie" symmetry group of the Riemannian geometry or metric of the space. All such 3-dimensional homogeneous spaces were classified and explicitly constructed by

Bianchi in his article. For context, our common notion of flat Euclidean 3-dimensional space has a geometry which is invariant under the translations of points of space (parametrized by 3 numbers: the translation amount in each of the 3 Cartesian coordinate directions) as well as under the rotations about any fixed point in space (parametrized by 2 angles which pick out the direction of the axis of the rotation plus one angle for the amount of rotation about that axis, both separately 3-dimensional Lie groups. The translation symmetry makes Euclidean space a homogeneous space, and the rotational symmetry about any point makes it also an isotropic space as well. The popular notions of an open, flat or closed spatially isotropic and homogeneous ("Friedmann-Robertson-Walker") universe depending on the value of the average mass density of matter in the universe compared to a certain critical value includes this flat Euclidean space as its simplest example (the flat space model), while the closed model space has the geometry of a 3-dimensional sphere within 4-dimensional Euclidean space and the open model space is a hyperbolic analog of that sphere, both of which are also spatially homogeneous and isotropic.

After my initial acquaintance with GR and cosmology as an undergraduate, I went on to become Abe Taub's last doctoral student at UC Berkeley (again in theoretical physics although he was in the math department) where I improved my knowledge of Lie groups, differential geometry, and their roles in mathematical cosmology, launching my long years of study in that field. Decades later when Abe died of old age in 1999 I learned of the rich history of differential geometry at Princeton that has been a fundamental tool for general relativity since its birth. Again by chance I discovered at the Princeton math/physics library an inaccessible paper-based oral history project in which he had participated centered on the Princeton mathematics community during the decade of the 1930s when Einstein was drawn to Princeton, and as a service both to Princeton and the world community, converted that enormous project into a website accessible to all.

2 Spatially homogeneous cosmology

Of course my work involves considerable specialization to follow, like any highly specialized field. While I have not produced any earth-shaking new results, I think my body of work has made a valuable contribution to the many others whose combined efforts together have advanced understanding in a difficult branch of mathematical physics. I became one of the leading experts in this field during the roughly 15 years that it was my primary focus. My first articles revealed how Lie groups and related advanced mathematical techniques were essential in understanding the consequences of symmetry in mathematical cosmology, and later articles built upon these essential ideas to further illuminate the dynamics of spatially homogeneous cosmological models.

Einstein's equations are a highly nonlinear set of partial differential equations which are very difficult to solve without additional assumptions usually involving symmetry. Due to the equivalence of points of space, Einstein's equations for spatially homogeneous cosmological models essentially reduce to the time-dependence of the geometry at a single spatial point, and hence become ordinary differential equations, which though still complicated, are much more amenable to study and to solution. These models, in addition to modeling deviations from the observed highly homogeneous and isotropic universe we observe on the large scale, have also served as a mathematical testing ground to better understand features of general relativity, the current theory of gravitation that fits experimental and observational evidence very well. In addition, they have played a role in exploring attempts to quantize gravitation and join it to the remaining fundamental forces of nature.

One of the most mathematically beautiful aspects of spatially homogeneous cosmology is the fact that is equivalent to the classical mechanical problem of a particle moving around in a force field with a symmetry group, and hence all the intuition and theoretical tools that physics has developed to treat that problem transfer to the mathematical cosmology problem. We all know about the conservation of linear momentum (due to the translational symmetry of ordinary space) and the conservation of angular momentum (due to the rotational symmetry of space). These conservation laws help simplify the description of particle motion. In a similar way the symmetry of spatially homogeneous cosmological models leads to a symmetry of the corresponding particle motion problem which simplifies its description, though still very complicated. This fundamental idea underlies much of my work in studying mathematical cosmology. I started out working alone, but eventually joined forces with a Swedish relativist Kjell Rosquist to whom I was introduced by Malcolm MacCallum in London where Kjell was a postdoc at the time. Our complementary skill sets allowed us to go further together, and soon his brightest student Claes Uggla became part of our collaboration, and he went on alone to extend our ideas to the more general setting of cosmological models without symmetry.

3 Spacetime splitting applications

Meanwhile my part time research life in Rome led to exploring other aspects of general relativity there with a student Paolo Carini, soon after joined by another student Donato Bini. One of the big conceptual advances made by Einstein was the idea of joining space and time together into the single arena of spacetime. However, we can only experience spacetime through the separate components of space and time that we experience as we travel through it. The measurement of physical quantities depends on the motion of the observer. Different observers in relative motion see things differently. In relativity energy (a scalar quantity) and momentum (a vector quantity) are joined together to form a single spacetime vector and in the same way that we can project an ordinary vector along different directions (basically just an extension of trigonometry), an observer is characterized by a spacetime velocity vector in spacetime which characterizes the direction of her motion in spacetime, and various spacetime quantities like the energy-momentum vector can be projected along the observer direction in spacetime to yield the individual quantities familiar from the world view of pre-relativity physics, like energy and momentum. In other words Einstein joined together energy and momentum as a single spacetime object, but each observer at a point in spacetime separates them apart again to interpret them as individual quantities. Thus we have a relativity of observer observations, each of which splits spacetime in a different way.

The familiar inverse square gravitational field of a point mass at a certain fixed location is the simplest model for calculating say the motion of the planets in the field of the sun within Newtonian gravitation. However, in Newtonian gravity, space and time are separate and noninteracting. In the corresponding model within general relativity, there is no preferred space and time in general. One must reintroduce them by specifying a family of virtual observers moving in the spacetime. This is sometimes referred to as a reference frame. For example at each point of space in the Newtonian model we can imagine what an observer fixed there would measure, say for the gravitational force on test masses located there. These observers, while fixed in space, are moving in time within the corresponding spacetime model, but in that general relativistic model, one is free to choose many different observer families which separate out space and time in different ways due to their distinct motions—and in a general complicated time-dependent gravitational field there is no preferred way of picking these observer families or reference frames.

Of course the point mass model has a lot of symmetry that allows one to adapt the observers to the geometry so that one can establish a direct correspondence between the quantities of the Newtonian theory which also governs our own world view and those of the general relativistic model. This allows one to interpret the more complicated GR model in terms of intuition from Newtonian theory. Centripetal acceleration and centrifugal forces are two such ideas that make it easier for us to wrap our brains around how gravity acts on particles; a circular orbit around the sun, for example, is said to have a balance between the inward gravitational force and the outward centrifugal force, which easily enables one to calculate the speed necessary to maintain such an orbit (easy if you have elementary knowledge of how physics works). But in the spacetime view of gravitation there is no gravitational force—particles simply follow the shortest path in spacetime, guided by the spacetime geometry. The introduction of a family of observers in spacetime enables one to reintroduce generalizations of the Newtonian concepts of gravitational and centrifugal forces that enable us to extend our previous world view to the more advanced spacetime setting of general relativity. In so doing, one finds a clear analogy with the theory of electromagnetism, which unifies the electric and magnetic fields into a single spacetime field. In the reverse way the single spacetime gravitational field when interpreted in terms of a spacetime splitting by a family of observers leads to identifying not only analog of the Newtonian gravitational field which behaves similar to the electric field in that it is generated by mass like the latter is generated by charge, but also a new aspect of the relativistic gravitational field which is analogous to the magnetic field and which is generated by rotating mass in a similar way to the generation of a magnetic field by rotating charge. These two fields were dubbed the gravitoelectric and gravitomagnetic files. The gravitomagnetic field is very small in comparison to the gravitoelectric field even for the rotating earth or sun, but is measurable. The Stanford University Gravity Probe B project (GP-B) which lasted over four decades has been the first to measure this directly (through extremely precise measurements of gyroscope precession in a polar orbiting satellite), but its presence has been inferred in the more extreme gravitational fields of relativistic astrophysical objects.

Paolo Carini got his PhD at Stanford in the GP-B group and through the close ties of the Rome group and our work together on these ideas, I was fortunate to have been able to participate in the GP-B satellite launch in 2004. Although Paolo had left research, our collaborator at the time Donato Bini has continued as an equal partner in our joint work of several decades that explores various aspects of spacetime splitting tools for understanding the gravitational field of compact objects.

4 Recent Work

Finally after many decades one of the ideas from the work of my postdoctoral period came back to the forefront involving the implications of spacetime splitting for spatially homogeneous cosmology generalized to arbitrary spacetimes. This led to a recent joint paper with James York, Jr., my first postdoctoral advisor, on a topic of his which had catalyzed my first published paper on spatially homogeneous cosmology from that period. Indeed that overlap had been responsible for me landing the job in the first place. With another younger collaborator Christian Cherubini, Donato and I have explored another cosmology problem, revisiting some very old ideas in a completely new light, enabling us to give a catchy title to our resulting article (Electrocardiogram of the Mixmaster Universe).

Our most recent work with another junior partner Andrea Geralico and a few other coauthors explores the general relativistic generalization of the Poynting-Robertson effect that was first studied in Newtonian gravity by one of the physicists whose name remains alive in electromagnetic theory (Poynting) and which was later refined essentially using special relativity and incorporating small general relativistic corrections by H.P. Robertson in 1937, a relativist who was my own PhD advisor's advisor at Princeton at that time (and therefore my academic grandfather so to speak). We have finally promoted the calculation to the fully relativistic setting for possible applications around relativistic astrophysical objects. The Poynting-Robertson effect is relatively simple to explain. Suppose a grain of interspace dust is orbiting the sun in a circular orbit. Due to the outward radiation pressure of the solar radiation it experiences a small outward force, but by absorbing the energy from the solar radiation which arrives from the sun slightly in the forward direction as seen by the particle because of the aberration of light, and then isotropically radiating that energy in its own "rest frame" by cooling down, there is a slight backwards effective force opposite to the direction of motion along the circular orbit. This acts as a drag effect to slow the particle down, causing it ultimately to slowly spiral inward to the sun. Of course for particles around compact objects with strong radiation flowing outwards in strong gravitational fields, this effect can be exaggerated, requiring a general relativistic calculation to describe. Like many such problems which are difficult to address in their full setting of complications, our problem is a toy model aimed at trying to understand how this effect might have consequences for real astrophysical objects under extreme conditions.