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GRAVITOELECTROMAGNETISM: RELATIVITY OF SPLITTING FORMALISMS

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ABSTRACT
Some brief remarks are made regarding the many possible approaches to splitting spacetime within general relativity and the subsequent definitions of related gravitoelectric and gravitomagnetic spatial gravitational force fields.

1. Introduction

Recent years have seen increasing use of the words *gravitoelectric* (GE), *gravitomagnetic* (GM), and *gravitomagnetism*, from which it is natural to introduce also *gravitoelectromagnetism* (GEM).[1] This terminology arises from an analogy with electromagnetism which becomes evident when one splits spacetime into space plus time for general relativity or other metric-based gravitational theories, introducing new metric variables and differential operators based on this splitting. However, a number of distinct splitting approaches can be taken for the fully nonlinear theory, all useful in certain contexts, while gravitoelectromagnetism is usually discussed only in an approximation scheme without much regard to the various splitting choices. In order to explore gravitoelectromagnetism carefully, one needs a single mathematical framework which can handle all of the different splitting choices and describe their interrelationships. A careful description of the fully nonlinear situation also helps to clarify the structure of the analogy with electromagnetism that occurs for linearized approximations.

One particular result of this analysis is a clean, conceptually simple, fully nonlinear derivation of the test gyroscope spin precession formula which occurs in discussions of the dragging of inertial frames. The usual linearized derivations, though entirely sufficient for the small effect to be measured in actual experiments, are rather confusing because of the mixing of the theory and the linearization which obscures the mathematical and physical structure of the problem.

The introduction of a sufficiently general description also has the advantage of putting many different results obtained over the past half century into context. For

*and International Center for Relativistic Astrophysics, University of Rome, I-00185 Rome, Italy example, most of us are somewhat familiar with the initial value problem for the ADM splitting of spacetime based on a family of spacelike hypersurfaces,[2] but the corresponding problem for the splitting of Landau and Lifshitz[3] based on a timelike congruence is almost unknown. In fact the exact solution industry for stationary spacetimes studies exactly this problem without the difficulties that breaking the stationarity symmetry introduces, since only the initial value problem remains of the Einstein equations in that case.

Extending the powerful language of the ADM splitting to the Landau-Lifshitz splitting helps bridge the gap between them and see connections which are not normally apparent. Each of these in turn are related to the partial splitting of spacetime based only on a unit timelike vector field, as used by Ehlers, Hawking and Ellis.[4] Unfortunately most of us tend to be married to a given approach, limiting our ability to treat certain problems for which an alternate approach may be more appropriate. The development of a unified language encompassing all of them allows one the option of choice.

2. A Matter of Principle

Usually gravitoelectromagnetism is discussed within the context of post-Newtonian theory, which is only natural since most realistic self-gravitating systems can only be analyzed through approximation schemes. However, there is some advantage to understanding the foundations of this approach in the fully nonlinear theory even though less realistic systems are amenable to analytic treatment. One reason is that various contructs which are indistinguishable in practice in the post-Newtonian theory have distinct and clearly interpretable meanings in the full theory and hence some ambiguity is removed.

Furthermore, the major differences that occur in different ways of splitting spacetime happen only when the family of test observers used in splitting the spacetime has nonzero vorticity, or rotation as it is more commonly referred to. "Rotating" spacetimes continue to cause confusion today, even in the context of rotating coordinate systems in Minkowski spacetime. It is therefore important to have a more powerful mathematical characterization of the arena in which this rotation occurs and see more clearly the connections with our nonrelativistic intuition about centrifugal and Coriolis forces.

Some of the confusing issues that arise from the discussion of gravitoelectromagnetism in the post-Newtonian approximation are 1) Is there a gravitational Thomas precession? 2) How can one distinguish a "true" gravitomagnetic field from an imposter due to a poor choice of test observer family? To some extent these questions arise from the lack of an unambiguous mathematical language with which to describe these problems. For example, "the gravitomagnetic field" is not precisely defined allowing different people to have different ideas of what it is supposed to represent. One needs qualifiers to clarify this controversy about a "true" gravitomagnetic field. This question is the analog of the local equivalence of acceleration and a gravitational force (due to a gravitoelectric field) but is a nonlocal one which depends on the family of observers rather than a single observer as in the elevator scenario. The simplest definition of gravitomagnetic field is a kinematical one, naively identifying a part of the splitting of the spacetime geometry, depending of course on the method of splitting. A "true" gravitomagnetic field would correspond to a dynamical definition relating the kinematically defined field to the "true" angular momentum of the source of the gravitational field through the field equations.

3. Splitting Spacetime

On the most general level, gravitoelectromagnetism is kinematically the natural generalization of special relativistic discussions of Lorentz transformations between global inertial reference frames and observer measurements of spacetime quantities to the arena of curved spacetimes where one is forced to consider all of this machinery independently at each spacetime point (apart from differentiability considerations). Once a family of test observers covering the spacetime is given, a metric based gravitational theory can be decomposed into three components through the introduction of a gravitoelectric vector field generalizing the Newtonian gravitational field, a gravitomagnetic vector field incorporating the new feature of local rotation into the field equations, and a spatial metric (SM) field reflecting the tensor character of gravitation

$$GEM = GE + GM + SM \ .$$

In fact the gravitoelectromagnetic vector fields reflect kinematic properties of the family of test observers (acceleration and vorticity respectively) while the spatial metric field describes the geometry of the space of observers, i.e., relative distances and angles. Dynamically, the gravitational field equations expressed in terms of these splitting fields provide the nonlinear generalization of Maxwell's equations for electric and magnetic fields alone to the triad of fields of the nonlinear case. One can introduce transformation laws for the nonlinear case as well, generalizing the well known laws for the electric and magnetic fields.

All of this can be done in the general setting of the "congruence approach" of Ehlers, Hawking and Ellis. Introducing analogous potentials for the gravitoelectromagnetic vector fields requires further structure corresponding to the Landau-Lifshitz or ADM approaches. These matters will have to be discussed elsewhere.[5]

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