2011 Outstanding Faculty Research Award Lunch Talk
bob Jantzen, November 9, 2011

My life combining differential geometry, general relativity and Italy

Father Peter, Father Ellis, Dean Linney, Dean Lindenmeyr, and fellow colleagues,

I am very honored to have been chosen for this award for which I thank the committee which selected me, and my colleagues Tim Feeman and Jesse Frey for nominating me, and together with my chair Doug Norton, for convincing me to go ahead with the process, and finally Fritz Hartman for a telephone call nearly 30 years ago about a missing transcript in my job application that perhaps got me invited to campus to land this job. Since this is supposed to be a brief talk, I thought I would try to sketch the human side of the equation that brought me here before you today.

20 minutes is barely enough time to scratch the surface of the web of human relationships which connected me up with the study of general relativity, Einstein's theory of gravity introduced 95 years ago in 1916. It certainly isn't enough time to do more than give a hand waving idea of what exactly I have been doing myself in this field, but perhaps more interesting is the social/historical setting in which it has taken place.

I grew up in a small village 90 miles north of Princeton where Einstein spent the final decades of his life. He arrived at Princeton University in the mid 1930s escaping fascism in Europe, finding refuge in the initial home of the Institute for Advanced Study shared with the university mathematics department before its permanent buildings were ready a mile away from the main campus. My PhD thesis advisor Abe Taub was a graduate student in both math and physics during those same early years at Princeton, studying under HP Robertson of the Friedman-Robertson-Walker-Lemaitre cosmological models. These models are the simplest description of the universe as a whole on length scales where it appears to be both homogeneous and isotropic. Lemaitre, by the way, was a Jesuit. But let's get back to the cosmology stuff later on in this talk.

My tiny village high school had a graduating class of about 30 students, so it was not a very good place to start an academic life. My working poor parents sacrificed by borrowing money they did not have to pay the out-of-district tuition for me and my other 3 brothers to attend the neighboring town public high school with a class size of about 160 that offered enough of a foundation to propel me to Princeton University, but not before encountering a fascinating little book from my village public library in the 9th grade written by a husband and wife couple, Lillian Lieber the mathematician, and Hugh Lieber the artist who illustrated it with whimsical sketches, a book called *The Einstein Theory of Relativity*, written in an amusing verselike format that inspired me to eventually be open to learning about general relativity. The idea of someday understanding the mathematics of curved spaces was stored, and then at Princeton, which was in its golden age of general relativity when I arrived as an undergraduate in 1970, I just sort of fell into it by chance. The key idea of Einstein was to change the paradigm of bodies accelerated by the gravitational force to one of bodies simply following a so called geodesic path in the curved spacetime formed by the set of all points in space and time. Spacetime is the arena of all points in space, a 3-dimensional space, at all possible times, making the result a 4-dimensional space, and a geodesic path is simply the closest path to a straight line in ordinary space, in which the direction of motion is locked straight ahead in the local geometry. In a sense, freely falling bodies just follow their nose in spacetime trying to keep moving straight, like walking towards the same fixed point on the horizon of the Earth without knowingly changing direction moves you along a great circle on the spherical surface of the planet. The presence of matter, like the massive sun in the solar system, stretches the spacetime fabric around it so that the planets in trying to simply continue going in the same direction in spacetime, end up orbiting around the sun in long stretched out helices along the time direction centered on the path of the sun in spacetime.

John Wheeler, a famous physicist who jump-started interest in general relativity at Princeton in the late 1950s, had been a friend of Einstein in his final years. I was lucky enough to have had Wheeler for sophomore modern physics, and he had attracted an Italian physicist to work with him on the physics of black holes, a term popularized by Wheeler not long before my arrival there. The collaborator’s name was Remo Ruffini, who became my physics godfather and enabler for 32 consecutive years of summer visits to Rome. For a project in his course on differential geometry for general relativity, I volunteered to write down his dictated translation of a 90 some page article from 1898 by an Italian mathematician Luigi Bianchi on the classification of all possible homogeneous 3-dimensional curved spaces. This is the spot where I can plug the college language requirement: I had taken 3 years of Latin in high school and called it quits, but then was forced to take a language at Princeton, so I picked the easy one: Spanish. Three semesters and done. But Remo has notoriously never been able to manage his time and I ended up stretching my Spanish into elementary Italian to get the job done myself, with his help in correcting the rough spots. This mandatory language requirement made it very easy for me to later learn Italian when I spent a year as a physics postdoc in Rome, without which my long association with Italy would never have been possible and certainly not as enjoyable. 30 years later, that amateur translation was polished up with the help of a Polish physicist for publication in the journal *General Relativity and Gravitation*.

While two German mathematicians were instrumental in establishing the foundations of the differential geometry of curved spaces in the late 19th century---first Gauss, and then his student Riemann---it was a pair of Italians who really made the subject usable as tensor calculus, the next step after multivariable calculus. This came just in time for Einstein to realize with the help of his friend Marcel Grossmann that it was just the tool he needed to make a geometric theory of gravitation that respected special relativity, which all came together in 1916. These two Italian mathematicians were Ricci and his student Levi-Civita, at the turn of the 19th century. Although Italy is well known for style and food, it also has a strong mathematics and science tradition. Finally this year my translation of two of Levi-Civita's landmark articles on general relativity appeared in the journal *General Relativity and Gravitation*. And just a few weeks ago I submitted a paper coauthored with Remo completing the early work of Enrico Fermi in his general relativity phase at the beginning of his career in the 1920s. Fermi was one of Italy's most brilliant physicists and was responsible for the first nuclear fission experiments, which took place at the University of Chicago just before the famous Manhattan Project of WWII. I was involved in translating his early papers from Italian as well, which will appear for the first time in English within the year.

But what about me as a college student back at Princeton? It turns out that Einstein's closest friend at the Institute for Advanced Study and extremely famous in his own right, Kurt Gödel, one of the most important logicians of all time for his incompleteness theorems, had a side hobby in general relativistic rotating universes. I had started studying these myself as a junior, and knowing of Gödel's interest, Remo looked him up in the phone book one evening and called to arrange an appointment for me at his office at the Institute. Gödel had used Bianchi's work on 3-dimensional spaces in a pair of momentous articles he had written in 1949 about rotating universes, and was still following current work in 1973 when I met him. He pointed out some articles to read and got me on course for my senior thesis work at Princeton. Good timing for me since sadly he starved himself to death only a few years later.

This undergraduate research led me to graduate school at UC Berkeley with Abe Taub, who had returned to Princeton to visit the Institute just after World War II for a few years when Gödel had been doing his famous rotating universe work, and then Abe had systematically presented the mathematical structure of all universe models with homogeneous but anisotropic spaces, generalizing the isotropic Friedmann-Robertson-Walker-Lemaitre models that are more familiar in popular cosmology discussions. These came to be known as the Bianchi models, named after Bianchi's fundamental work on all these possible spatial geometries. I never did get to ask Abe whether Gödel's work had influenced him or not---he died in 1999 just as I was getting interested in this history, but by chance looking for materials about Abe at the math-physics library at Princeton, I learned of the existence of an oral history project about the remembrances of former mathematicians who had participated in the decade of the 1930s when the Institute for Advanced Study and the university math department shared a building together and many Europeans passed through fleeing fascism, like Fermi had been forced to do as well. This was also the early stages of the internet, and one thing led to another and I ended up taking this inaccessible 700 page paper document that I had found and converted it into a website at Princeton University with accompanying materials that made it available to anyone who was interested.

By the time I came to them, the Bianchi universe models were an important testing ground for trying to understand the complications of general relativity. Because of their high symmetry, the Einstein equations for these models were much more tractable than models with lesser symmetry---one could deal with simpler ordinary differential equations for the time evolution of the gravitational field of the universe alone instead of terribly complicated partial differential equations for both the space and time behavior of a more complicated model. Over the next two decades I made some modest contributions to this field. Einstein's equations for these universe models have a lot of beautiful mathematical structure due to their high symmetry, and I was able to help in bringing some of this to light.

And then I became interested in the gravitational fields of compact objects like the Earth or Sun or black holes, where again symmetry plays an important role, and another Italian mathematician/physicist served as a touchstone. Carlo Cattaneo had been active in general relativity at the University of Rome in the 1950s and had pioneered an alternative approach to splitting Einstein's spacetime back into space plus time. In fact many different approaches to this splitting had been taken by relativists in Europe and America. Remo never really appreciated what Cattaneo had tried to do, so somehow I got the job of figuring it out in the context of all these other approaches.

While Einstein had made an important conceptual leap forward to join space and time into a single 4-dimensional continuum called spacetime, we are creatures who can only experience this spacetime through 3-dimensional spatial concepts which evolve in time. To interpret the beautiful 4-dimensional descriptions of the gravitational field and the universe, we must interpret it in terms of our ordinary 3-dimensional experience. We must slice up spacetime into a successive family of 3-dimensional spaces of fixed times, and thread it by a family of world lines along which we measure the time elapsing at those points of space---in other words, we must split spacetime into space plus time: space, which is really a moment of time in our universe, and time which elapses at each point of space as measured by a hypothetical time-keeping device sitting there. This splitting is needed to relate the abstract geometry of motion in spacetime to our more mundane notions of acceleration and force, which are associated with time rates of change of spatial position.

In Newtonian gravity and nonrelativistic physics we think of the points in space around the Earth as not moving, and satellites which orbit the Earth in circular orbits under the balance between the inward centripetal acceleration taught in calculus or in high school physics and the inward gravitational acceleration towards the center of the Earth. On the other hand to stay in place at a fixed location in the space around the Earth, a rocket must have a rocket thruster firing away from the Earth to keep it from falling down, in order that it have zero velocity and zero acceleration with respect to points fixed in space. In contrast in the spacetime point of view of general relativity, the satellite in circular orbit is freely falling in spacetime and has zero spacetime acceleration, while the rocket sitting at a point fixed in space is considered to be accelerated in spacetime in order to resist its natural free motion towards the center of the Earth. Exactly the opposite assignments of zero and nonzero acceleration.

However, to match back up with the Newtonian concepts of force and acceleration, it is enough to consider the relative acceleration measured with respect to the accelerated points fixed in space---in each of the above points of view, the difference accelerations agree. The freely falling nonaccelerated circularly orbiting satellite appears to be accelerated with respect to the accelerated points fixed in space because the difference of their spacetime accelerations is nonzero. Subtracting the outward spacetime acceleration from the zero spacetime acceleration of the circularly orbiting satellite gives the inward relative acceleration that is what we associate with the acceleration due to the inward force of Newtonian gravity. The points fixed in space describe an accelerated reference frame in spacetime with respect to which the freely falling objects have a nonzero relative acceleration, and this enables us to directly compare the general relativity picture with the Newtonian gravity picture. Without confusing you any more, it suffices to say that relative motion is the key to interpreting the abstract 4-dimensional spacetime geometry in terms of our traditional view of the arena of pre-relativistic physics.

Besides relativistic corrections to the Newtonian force of gravity obtained in this way, general relativity has an entirely new physical effect due to the rotation of a massive body like the Earth. The rotation of the Earth within the surrounding spacetime pulls the nearby spacetime along with it slightly, like rotating a spherical ball in a viscous fluid where the cohesive forces at the surface of the sphere next to the fluid pull it along, with the effect lessening with distance from the surface. This is called the “dragging” of spacetime and can be directly measured with sophisticated gyroscopes. In Newtonian physics, the axis of rotation of a gyroscope on which no rotational forces act remains fixed with respect to space, that is with respect to the fixed stars. In general relativity a gyroscope freely falling in orbit around the Earth instead keeps its axis direction fixed with respect to the local spacetime instead of the distant stars, and since that local spacetime is itself slightly rotating as the Earth pulls it around with its rotational motion, the gyroscope axis will itself rotate with respect to the fixed stars. Of course this is a really small effect and very difficult to measure.

Only last year a 4 decades long big science project called Gravity Probe B or GP-B for short came to completion to measure this really tiny effect on gyroscopes in a sophisticated low temperature polar orbiting drag free satellite, the details of which are all explained for ordinary people on its website easily found with a search engine. Two years of data were taken to measure two separate relativistic effects due to general relativity. The gyro directions were compared to a guide star and their spin directions changed by these two effects by angles in perpendicular directions so they were easily distinguished. The first effect was due to the curvature of space alone along the circular orbit, and the other much smaller due to the dragging effect, whose accumulated change in direction over a year was equivalent to a total angle change equal to the angle subtended by a quarter at 40 miles, in other words, incredibly small.

To do one little gee whiz illustration of spacetime curvature affecting the direction of the gyroscope spin axis meant to be fixed with respect to the stars, suppose this circle is an orbit around the Earth and we pick a direction fixed with respect to those stars, see all the parallel arrows point in this same direction upwards. Next suppose we cut out a wedge and make a cone. The orbit is still a circle, but after moving around it one revolution, the final arrow is rotated with respect to the initial arrow by exactly the angle we cut out. General relativity bites away about 1 inch of the circumference of the low Earth orbit circle for the GP-B experiment, causing a very small change in the direction of the gyro spin during each orbit. This is the space curvature effect, the larger of the two effects.

After many years of data analysis taking into account some messy details that were not foreseen in advance, the larger space curvature measurement confirmed general relativity to a quarter of 1 percent, while the much tinier dragging effect was confirmed to a 20 percent margin of error. I was fortunate to be present at the launch site of the satellite one April weekend in 2004, although the wind did not cooperate so it was aborted at a few seconds before launch so we only got to enjoy the celebration party since I had to get back to teach. It went up the next day. My work in spacetime splitting helped clarify how these really tiny effects that it measured are aspects of the general theory which are reflected in the mathematical structure of very strong gravitational fields, where various analogies can be drawn with the theory of electromagnetism: the Newtonian gravitation-like field of a point mass is analogous to the electric field of a point charge, while the dragging effect of a rotating point mass is like the magnetic field of a rotating charge. However, there is not just one way of drawing the analogy, but instead a rich family of different ways which can be dubbed gravitoelectromagnetism, a big word for some simple ideas that involve complicated mathematics describing a relativity of spacetime splitting approaches.

To draw these remarks to a close, I have to admit I am an atypical member of our newly renamed Department of Mathematics and Statistics. An academic trained as a physicist but armed with the powerful mathematics of some of the most imaginative people from our past, enabled to dance around some of the abstract ideas of a theory which helps us understand our place in the universe, and connected by chance to a nation that is frequently in people's minds for food and design but also has contributed more than its share to mathematics and science. Thanks for your patience in trying to follow my rambling remarks that roughly track my serendipitous journey through a lifetime of dr bob academic experiences. And let me also give Brother Guy from the Vatican Observatory a plug for his talk at 4pm today in the Driscoll Auditorium on the topic *Creation Stories*.