[Introduction to]
Cosmological Models: 4 Lectures

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This is the origin of the later Introduction to Cosmological Models notes, from which these were cannibalized to fill out Part I.

- Part I: orthogonal coordinates on flat or constant curvature manifolds with metric, FRW geometry, simplest spacetime splittings, Gaussian normal coords, intrinsic and extrinsic curvature (January 1988)
(icm1.pdf: 37 pages, 1.163 MEG)

Pages marked page 35, 36, 37, 38, [40] were inserted in Part II at the end, together with the Lie derivative pages from the Lifshitz perturbation analysis:

- Part II: Symmetries and Lie groups (March 1988)
(icm2.pdf: 29 pages, 909K)

The original introduction pages contain the Symmetry Breaking in Cosmology diagram from which later notes developed:

- icm1984.pdf, 7 pages, 350K

COSMOLOGICAL MODELS: 4 lectures by Bob Jantzen

This is not an introduction to general relativity and does not presume a knowledge of general relativity. It is a survey of the subject of cosmological models, introducing some mathematical tools which are extremely important in all areas of modern physics. Four lectures are too many for a very superficial survey but too few for a detailed review. The compromise will be a guide that will hopefully stimulate an interest which you will be able to pursue afterword in much greater depth.

The large scale structure of the universe is described by very symmetric cosmological models. The small scale structure is of course very unsymmetric and is described in terms of small perturbations of the symmetric models, due to the complexity of the full nonlinear Einstein equations. The study of the large scale structure leads immediately to the theory of Lie groups and homogeneous spaces, while the study of the small scale structure leads to harmonic analysis on these spaces (involving representation theory). These techniques by themselves are very important in elementary particle theory, for example, but so too is their application in the description of the early universe where particle theory and cosmology necessarily join forces to explain the present universe and constrain the models of particle physics.

A knowledge of special relativity is assumed.

OVERVIEW

The most symmetric cosmological models are either spacetimes of constant curvature or have spatial sections of constant curvature. The nonflat members of this class of pseudo-Riemannian or Riemannian manifolds may be realized as pseudospherical hypersurfaces in flat pseudo-Riemannian manifolds of one greater dimension. These "pseudospheres" are transformed into themselves by the pseudo-orthogonal groups which are therefore the groups of motions of nonflat spacetimes or spaces of constant curvature.

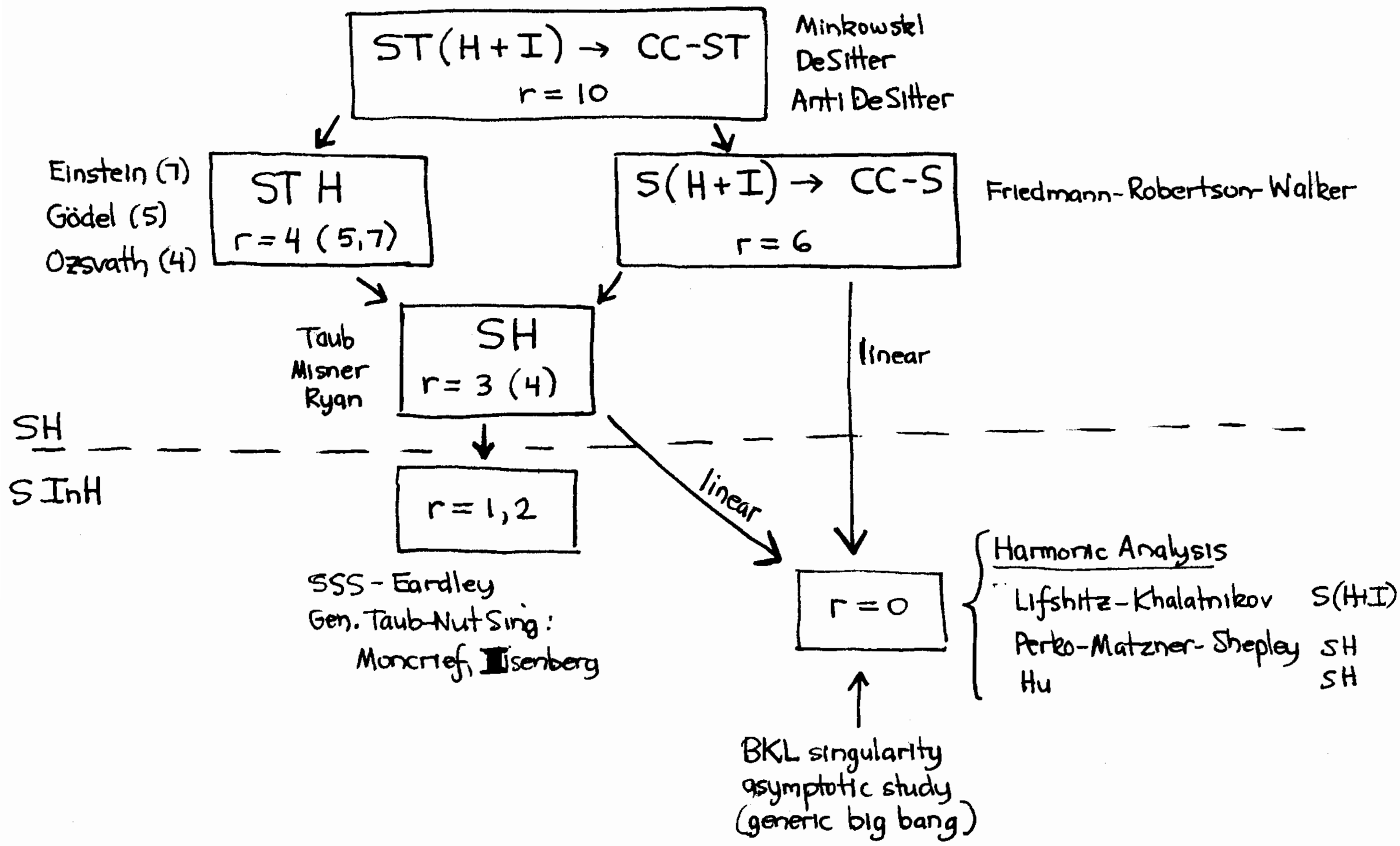
In each of these cases the geometry (or metric) is maximally symmetric in the sense that it is both homogeneous (all points equivalent) and isotropic (all directions equivalent, consistent with the causal structure of the geometry). Such spaces (in the generalized sense) are <u>homogeneous spaces</u>, namely spaces of the form G/G_x, i.e. quotient spaces of a group G (the full group of motions of the space) by a subgroup G_x (a subgroup which leaves a typical point x fixed). Relaxing the condition of isotropy leads to a much larger class of cosmological models which are characterized only by homogeneity in spacetime (spacetime homogeneous spacetimes) or in space (spatially homogeneous spacetimes). These spacetimes are invariant under 4-dimensional and 3-dimensional groups of translations respectively and the spacetimes and spatial sections respectively are themselves Lie groups since one may identify the group of translations with the space itself, exactly as in the familiar case of the abelian group of translations of R^3. One is thus led to study translation invariant metrics on Lie groups of 4 and 3 dimensions respectively. In fact the metrics of the constant curvature spaces are also closely related to such metrics on the pseudo-orthogonal groups.

Finally when one relaxes the condition of homogeneity, one is forced to consider the linearized Einstein equations for inhomogeneous perturbations of homogeneous models, due to the complexity of the full nonlinear equations. This naturally leads to the application of the theory of harmonic analysis on Liegroups and homogeneous spaces as a means of solving the perturbation equations. This is just familiar Fourier analysis in the case of an abelian group. The most important physical question relevant to this analysis is the formation of inhomogeneous structure in the universe such as the galaxies, clusters of galaxies and superclusters.

We will therefore begin with Euclidean and Minkowskian spaces, consider pseudospheres in these spaces and then generalize to n-dimensional flat spaces of indefinite metrics (to consider the DeSitter groups), leading to a description of the spaces and spacetimes of constant curvature and their associated symmetry groups. In order to go further, as well as to appreciate this material at a more than superficial level, abstract Lie groups and Lie algebras and the action of such groups on the spaces of constant curvature as homogeneous spaces will then naturally follow as well as the pseudo-Riemannian geometry of such spaces. The various cosmological models of decreasing symmetry will then be reviewed (the standard model, the inflationary model implied by GUTS, the spatially homogeneous but anisotropic models). Finally perturbation theory on such spaces will be described in terms of harmonic analysis on homogeneous spaces, if time permits.

SYMMETRY BREAKING IN COSMOLOGY

a rough summary of the symmetry types of relativistic cosmological models



ABBREVIATIONS

r = dimension of full group of motions

ST = spacetime

S = space (adjective: spatial; adverb: spatially) ?

H = homogeneity (adjective: homogeneous)

I = isotropy (adjective: isotropic)

InH = inhomogeneity (adjective: inhomogeneous)

CC = constant curvature

SSS = spatial self-similarity

REFERENCES

Bernard Schutz: *Geometrical Methods of Mathematical Physics*
Cambridge University Press, 1980
[₤ 30.000 Anglo American Bookstore (tel: 6783890)
1½ months individual order, 3 weeks group order]

This book is extremely useful as a basis for the modern mathematics on which most of the ideas in recent physics rely. I cannot recommend it highly enough. If I accomplish nothing else but convince you to buy this book which you may read at your leisure, I will be satisfied.

S. Weinberg: *Gravitation and Cosmology*
Best description of the standard model of the universe

L.C. Shepley, M.P. Ryan: *Homogeneous Relativistic Cosmologies*
The only book devoted to the study of relativistic cosmological models. Elementary level.

S. Hawking, GFR Ellis: *The Large Scale Structure of Spacetime*
Good summary of differential geometry, global structure of cosmological models.

C. Misner, K. Thorne, J. Wheeler: *GRAVITATION*
The book on general relativity. Good elementary introduction to the mathematics

Robert Gilmore, *Lie Groups and Lie Algebras for Physicists*
Wiley-Interscience, 1974
This is the best book I know of for an introduction to both Lie groups and Lie algebras from a physicist's point of view

F. Warner *Foundations of Differentiable Manifolds and Lie Group Theory*
More sophistocated, useful after basics are mastered.

N.J. Vilenkin, *Special Functions and the Theory of Group Representations*
good introduction to representation theory and harmonic analysis

R. Abraham, J. Marsden *Foundations of Mechanics*
dynamical systems involving invariant metrics on Lie groups (generalizing the rigid body system)