

DIFFERENTIAL GEOMETRY NOTES

BASED ON

UNDERGRADUATE

LINEAR ALGEBRA AND MULTIVARIABLE CALCULUS

Spring 91

Villanova University

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DIFFERENTIAL GEOMETRY : CLASS NOTES.

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The "brilliant" off the cuff lecture given on the first day motivating index positioning and conventions and the dual space concept on the one hand and flat and curved spaces and calculus on the other was not recorded. Perhaps I can reconstruct it later. For now, just remember that something preceded page 1 that makes starting up like this a bit more digestible.