DIFFERENTIAL GEOMETRY NOTES

BASED ON

UNDERGRADUATE

LINEAR ALGEBRA AND MULTIVARIABLE CALCULUS

Spring 91

Villanova University

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The "brilliant" off the cuff lecture

given on the first day, motivating index positioning and conventions

and the dual space concept on the one hand and

flat and curved spaces and calculus on the other

was not recorded. Perhaps I can reconstruct it

later. For now, just remember that something

preceded page 1 that makes starting up like this

a bit more digestible.