

240-3.
LECTURES ON ATOMIC PHYSICS
THIRD LECTURE (*)
THEORIES ON THE ORIGINS OF THE ELEMENTS

“Teoria sulle origini degli elementi,”
Conferenze di Fisica Atomica (Fondazione Donegani),
Accademia Nazionale dei Lincei (1950),
compiled by Prof. E. Pancini

All the known matter is made up of various chemical elements each present with a different abundance, so the problem arises, first experimentally and then theoretically, of understanding for what reason some elements are abundant, others rare.

The problem is first of all an experimental one and, not wishing to discuss the question in detail here, a few general considerations are enough to understand it. What we are trying to establish are the amounts of the various chemical elements which are, so to say, in the whole Universe or, at least, in a large part of it and, obviously, the result which we may expect to obtain depends to a large extent on the samples taken for the analysis. For instance, if it is possible to determine the relative amounts of oxygen, iron, hydrogen and the other elements present in the part of the terrestrial crust which is approachable by our direct observations, one will get for each of them a definite relative abundance. But if, on the contrary, one determines for instance, the percentages of the same elements by analysing the meteorites, a different distribution of the elements with respect to the one found in surface rocks on the Earth will be discovered.

Therefore the problem, rather than a problem of chemical analysis, is essentially a problem of the selection of the samples to analyse.

Obviously, the question is not a new one; the data which will be presented here have been obtained in rather recent research by Harrison S. Brown,¹ of the University of Chicago, who has extended, enlarged and perfected the results of Goldschmidt.² The data have been obtained by analysing a large quantity of samples and this assures their reliability because data obtained from a particular sample

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¹*Rev. Mod. Phys.* **21**, 625 (1949).

²V.M. GOLDSCHMIDT, *Geochemische Verteilungsgesetze der Elemente und der Atomarten*, IX, Oslo, 1938.

display the special characteristics of it instead of what can be considered the cosmic distribution of the elements.

It is noteworthy the fact that, in spite of this observation, the conclusion of these analysis is that, if the selection of the samples is made with suitable attention, the results are highly uniform even if derived from materials of very disparate origin. For instance, in some favorable cases, it is possible to assign the ratio of the cosmic abundance of two elements with a precision of the order of the 1 or 2%.

Note that the measures of the abundances of the elements performed on the terrestrial crust, even if of utmost practical importance, have a rather limited theoretical importance. In fact the terrestrial crust, indeed all the material which constitutes the Earth, during the geological ages, has been subjected to a deep chemical separation so that one could obtain significant results only through an analysis of samples taken in zones which go from the Earth's surface to its center and this is, obviously, impossible.

Luckily, this impossibility of taking samples from the interior of the Earth can be circumvented by studying the composition of the meteorites which, in the opinion of the experts, turn out to be samples taken from various zones of missing planets. Thus, if the Earth, due to a cosmic catastrophe, were to break up, the meteorites coming from its crust would be essentially made of iron or, more precisely, of an alloy consisting mostly of iron and then nickel and then, to an ever lesser extent, other elements.

In effect a statistical analysis of the meteorites which arrive on the Earth (and the meteorites which arrive on the Earth are really of these two kinds) indicates that the ratio between the amount of matter from stone meteorites and iron meteorites is not very different from the ratio between the stone part and the iron part of the Earth which results from the investigation in depth through seismic waves.

The research of which we are speaking has been carried out mostly by a painstaking collection of a large number of meteorites and performing extremely accurate quantitative analyses of them. It's important to remark that the problem of making these analyses is much less simple than it might seem because most of the elements, indeed, as we shall see, almost all, are so rare as to be present in amounts of few parts per million or even less. Thus one of the greater difficulties of the problem is that of finding the means of performing quantitative chemical analyses of extreme sophistication. To overcome this difficulty one has been even obliged to use (at least in the recent studies performed at the Chicago University) nuclear reactors for irradiating the material under examination with the aim of observing the resultant characteristic activities of the elements of interest. Therefore the identification of the elements, in this way, is realized essentially through radioactive rather than chemical methods. By those means one has succeeded in analysing the material arriving on the Earth in the shape of iron and stone meteorites and, taking suitable averages of the data so obtained, a table has been constructed which for most of elements will coincide with other data of completely different origin as, for instance,

those obtained through the spectroscopic observation of the stellar atmosphere. In this way one has an indication that the matter which constitutes the meteorites is not substantially different from that which constitutes stellar atmospheres. As a matter of fact there are some remarkable exceptions, but easily understood: for instance there are some elements in the meteorites which are practically missing, or present in an amount smaller than the one expected. This is the case of the noble gases which are present both in the Earth and in the meteorites in an amount largely smaller than that corresponding to their cosmic abundance because in the process of formation of the planets they have not been kept inside. Another exception even more notable is that of hydrogen, but also this exception is accounted for by arguments of the same kind of those put forward for the rare gases.

Facts of this kind make it clear how the results obtained from the analysis of the meteorites (from which we take most of the quantitative data on the cosmic abundances of the elements) should be revised through a careful chemical discussion which, on the other hand, is unfortunately almost completely arbitrary since it involves assumptions about the process of formation of the meteorites themselves and the characteristics both chemical and physical of the environment where they have been formed. In short the analysis of meteorites, element by element, must be integrated together through chemical considerations of a theoretical nature which allow us to decide if the element in question has retained its cosmic proportions in the meteorites.

Besides meteorites also the stellar atmospheres have been investigated (through spectroscopic analysis) and, in part, the matter clouds of the interstellar space through the analysis of their absorption spectrum. These data are, nevertheless, extremely limited and must be used only as additional ones. But the fact that is anyway remarkable is that all these data (taking into account the quoted exceptions which, in any case, can be justified by very convincing arguments) coming from the analysis of quite different celestial objects as the stellar atmosphere, the interstellar matter and the meteorites all match very well. On the contrary, the data which come from the Earth's crust vary significantly because, as we have said, the Earth's crust is not a faithful specimen of what can be considered to be the material which constitutes the Earth.

That said it is interesting to consider the table reported here (Table 1) in which the numbers represent the relative abundances of the various elements.

These data are extracted from the papers of Harrison S. Brown which can be considered the most up-to-date; the numbers listed in the table refer to some of the most significant elements and are sufficient to point out some strange features of the behavior of the relative abundance of the various elements as a function of the atomic number. They represent the number of atoms of each element present, on the average, in the cosmic matter for every 10^4 atoms of silicon 14.

When analysing this table it is convenient to begin at hydrogen, which is not only the simplest one of the elements, but also the most abundant: the number

	A	Z	Atoms per 10^4 atoms of silicon
H	1.01	1	3.5×10^8
He	4	2	3.5×10^7
Be	9.02	4	2×10^{-1}
C	12.01	6	8×10^4
O	16.00	8	1×10^5
Si	28.06	14	10^4
Cl	35.46	17	2.5×10^2
Mn	54.93	25	1×10^2
Fe	55.85	26	2.6×10^4
Co	58.94	27	1.6×10^2
Ni	58.69	28	2.0×10^3
Cu	63.57	29	7
Ga	69.72	31	5×10^{-3}
Sr	87.63	38	10^{-1}
Cd	112.41	48	2×10^{-2}
Cs	132.91	55	10^{-2}
Pt	195.23	78	10^{-1}
Pb	207.21	82	4×10^{-3}
Th	231.12	90	10^{-2}
U	238.07	92	3×10^{-3}

of its atoms present in the cosmic matter for every 10^4 atoms of silicon amounts to three or four hundred million. After the hydrogen, both in the periodic system of the elements and in the scale of the abundances, there is helium whose relative abundance is, on our scale, 35 million. For the elements which follow helium, the relative abundance decreases very rapidly to extremely low values: lithium, beryllium and boron are extremely rare: for instance, the relative abundance of beryllium is two tenths (that is, there are 50,000 atoms of silicon for every atom of beryllium). As one can see, between helium and beryllium there is a jump on the order of one hundred million.

The other light elements which follow the three quoted above in the periodic table have abundances slightly different from that of silicon: to carbon, for instance, an abundance of 2×10^4 must be attributed. Immediately after oxygen heads upwards: 10^5 . It is, after hydrogen and helium, the most abundant element as regards the number of atoms (not the weight).

Proceeding on this scale one finds abundances on the order of few units until one arrives at iron which has a considerably high abundance: 2.6×10^4 . Then cobalt: 2.6×10^2 , nickel: 2×10^3 , and continuing on in the order of the periodic table, at this point the abundance begins to decrease rapidly and does not rise any more.

From gallium on until uranium the abundances oscillate more or less irregularly between one tenth and one hundredth. A slight exception is lead which has a little higher abundance, but one might think that the amount of lead is increased due to the decay of the radioactive substances which are located immediately over it. Another exception, in the opposite sense, is uranium but one can think it became impoverished owing to its radioactive decay.

All these arguments will probably assume more clarity if we represent in a diagram (Fig. 1) the relative abundance of the elements as a function of the atomic number. From this diagram one can see that immediately after the peak represented by hydrogen and helium there is a tendency to exhibit, though with high regularity, a decreasing feature of the relative abundance of the elements. So that one who wanted to draw a curve through these points, neglecting the irregularities, could draw the curve shown in Fig. 1. And, if one wanted to trust that, having accounted for the exceptions, this curve represents with good approximation the relative abundance of the elements, one should also conclude that the relative abundance of each element is one of its essential characteristics like, for instance, its atomic number, its energy of formation or its mass. Then one forms the impression that the relative abundance of each element is really a property of its own, connected, as is obvious, both with the other properties of the element and with the mechanism, quite unknown, through which the element has been formed.

Obviously, in a discussion of this kind one must take into account the abundance of the different isotopes of each element, but this is not a complication of the problem since the relative abundances of the isotopes of each element are known and rather constant: so if we know the abundance of each element it is a question of trivial arithmetic to calculate the abundance of the isotopes.

Also when studying the relative abundance of the isotopes one will notice some regularities that are worthwhile to call attention to because we shall come back to them below.

In Fig. 2 a diagram of isotopes is shown with the number of protons which constitute each nucleus given along the abscissa and the number of neutrons along the ordinate: as one can see the various elements are distributed in a region which initially has the direction of the bisector of the axes and then turns upwards (and this means that in the nuclei of low atomic number the number of protons equals that of neutrons whereas for the high atomic numbers the percentage of neutrons is, more and more, higher than 50%). Almost always one finds that in the lower part of the periodic system — that is for the light nuclei — the more abundant isotopes are those richer in protons or, what amounts to the same thing, poorer in neutrons; then there is a transition zone and finally in the higher part one observes quite the opposite tendency: the more abundant isotopes tend to have more neutrons than protons.

Obviously the idea of justifying all these facts, that is to justify the abundance of every single element and, for each element, the abundance of its isotopes, is,

certainly, an extremely ambitious program and constitutes a problem whose solution is assuredly very far off. Nevertheless recently there have been attempts in this direction but with quite unsatisfactory results. This fact does not exclude that they are extremely interesting in the sense that they represent an attempt at research in the direction which most probably will be one of the most important in the future. On the other hand it is obvious that if the solutions obtained to date are not satisfactory one cannot exclude that in the future one cannot make conclusive steps along this road.

One of the most natural hypotheses that has been formulated, from long time and by various people, is that the elements we find in nature are the result of a process which bases itself on a kind of a chemical or, as one says, superchemical equilibrium. In other words one can ask if it permissible to imagine that if we put the constitutive elements of the chemical elements in a cauldron, that is protons and neutrons, and then heating it all to a suitable temperature and finally, when this matter is, so to say, well cooked, cool it suddenly, one can obtain a mixture of elements which looks like that which seems to us to constitute the Universe.

Many attempts in this sense have been made but the results obtained are indeed not very convincing. Obviously the temperatures and the pressures in this kind of cauldron must be thought to have rather amazing values if one wants to obtain results which do not disagree with the experimental data: for instance, the temperature should be about 10 billion degrees and the pressure about one million grams per cm^2 . The necessity of such temperatures and pressures can be understood without difficulty if one considers that the temperature must be very high to bring forth, in a conspicuous manner, nuclear reactions, and that the pressures must be also be very high to have the possibility of forming very heavy nuclei. In fact if, at such high temperatures, the pressure were not proportionally high, all the nuclei consisting of many particles would disintegrate and the possibility of the existence of heavy nuclei present in nature would not exist.

On the other hand, the fact of the matter is that starting from such hypotheses one does not succeed in obtaining a distribution of elements which looks very much like the real one: for instance, the relative abundances of various isotopes turn out to have a random distribution absolutely different from that observed experimentally.

The more recent theories are based, instead, on a rather different scheme: we shall limit ourselves to speak here about only one of them, that which, in our opinion, is the most interesting one even if it cannot be considered in any way to give a satisfactory explanation of the facts. This theory is due chiefly to Gamow who, being a joker as everybody knows, joined with two other physicists, Alpher and Bethe, with the aim, perhaps, of playing with the fact that the three names, read in the American way mangling the words, sound like the first three letters of the Greek alphabet. As a matter of fact, the essential contribution of the theory of which we are speaking was given by Gamow and in part by Alpher: Bethe, instead, appears to have been associated only to complete the play on words.

Anyway, the theory can essentially be divided into two parts. The first is based on the observation, in reality not new, that there is the possibility of forming elements, even when the temperature and pressure do not assume such amazing values as those quoted above, provided that one conjectures forming the elements through successive additions of neutrons. Without dwelling at the moment on an explanation of the origin of these neutrons, let us try to give an idea of the way in which this formation can take place.

Let us still refer to a P-N diagram (Fig. 2) in which each element is represented by a point whose abscissa is equal to the number of its protons and whose ordinate is equal to the number of its neutrons. As we have already said, all stable elements are located in a well defined zone.

If we now assume that we submit a certain element to a "bath" of neutrons it may happen that its nuclei capture one of these neutrons. Thus, if the composition of this nucleus is represented by the point A of Fig. 3, after the capture the new formed nucleus will have a composition represented by the point B which is obtained by taking from A a step upwards (in fact N is increased by one and Z remains constant).

The new nucleus will be able, in turn, to absorb another neutron and produce an element represented by the point C and so on, until one ends up going out of the zone of stable elements. The newly formed unstable element will evidently be beta radioactive and then will disintegrate through a beta process which is a change of a neutron into a proton: the new representative point will therefore be obtained taking a step downwards (decrease of a neutron) and a step rightwards (increase of a proton).

If now there are still neutrons present, the nucleus so formed will be able to absorb another neutron, then another neutron and after it will emit a beta ray; and in this way little by little we will climb up the slope of the stable elements. Thus little by little very heavy elements are formed through a mechanism of successive additions of neutrons to light nuclei assumed to be pre-existing.

At this point, if one wants to be ambitious (and, as we shall see, Gamow puts forward demands still more ambitious than these), one can even intend to explain the formation of all the elements starting from only neutrons.

Let us assume, in fact, that in a region of space, at a certain instant, are contained some neutrons. As is known, the neutron is not a stable particle, on the contrary its life time is quite short (it has not yet been measured very well but it cannot be appreciably different from $15'$) and therefore after about ten minutes, half of the neutrons will be decayed producing as many protons. But neutrons and protons have a certain affinity and the neutrons tend to latch onto the protons in this way forming nuclei of deuterium. In this way, starting initially from only neutrons, through their decay and association with the generated protons, it will be possible to form the first light nuclei and then, from them, with a process of the kind described above, one will arrive presumably at the formation of the heavy

elements.

Gamow has made an attempt at investigating this model (or better put, a model which looks like this) from a quantitative point of view. Of course for a quantitative investigation it is necessary to introduce data on the probability of capture of neutrons by a given element, because it is this probability which essentially determines the speed of the process of this phenomenon. Now, much data is available on the capture of slow neutrons, but presumably phenomena of this type happened at a temperature high enough to advise taking data regarding the capture of rather fast neutrons — and Gamow has taken data of this type. In Fig. 4 we report the cross sections for the capture of fast neutrons as functions of the atomic weight.

Gamow, simplifying (maybe too much) that what the experimental results really give, assumes that these cross sections for the capture of neutrons increase linearly for values of the atomic weight between 0 and 100 and then remain constant as indicated by the broken line depicted in Fig. 4. If we observe the figure and take into account that the scale is logarithmic, we can judge how much Gamow's schematization is strong: anyway it is convenient to follow Gamow's reasoning till to the bottom before criticizing it.

Therefore let us assume, for the time being, that the cross sections are really the ones Gamow claims. In this case we can plainly write down the differential equations describing how heavier elements are successively formed. Let us call N_a the number of atoms with atomic weight a ; the derivative of this number with respect to the time will depend on two terms: one which represents the increase in the number of atoms of weight a due to the aggregation of atoms of weight $a - 1$ (and this will be a positive term proportional to N_{a-1} , to the cross section σ_{a-1} of the element $a - 1$ and to the flux $\Phi(t)$ of the neutrons). Then there will be a negative term which in the same way represents the decrease of N_a due to the absorption of neutrons which changes the atoms a into atoms $a + 1$. In a formula

$$\frac{dN_a}{dt} = \Phi(t) (\sigma_{a-1}N_{a-1} - \sigma_a N_a) \quad (a = 1, 2, \dots, 238) . \quad (1)$$

Equations like this must be written for every value of a and a system will be obtained which we can solve, at a fixed neutron flux, deriving the way in which the abundances of the single elements evolve in time.

Now, the most significant result (which could be even more significant if the curve of the capture cross sections assumed by Gamow were a more faithful representation of the experimental facts) is that, owing to the peculiarity of this curve — namely the fact that for a certain atomic weight the tendency of the cross sections to increase stops suddenly — by assuming conveniently the time and the flux of neutrons, one finds a distribution of the abundances of the elements of the type represented in Fig. 5 and which is not very different from the experimental one (Fig. 1).

Of course the result one obtains depends on the time interval we choose in the sense that, if we fix a certain flux of neutrons, the material must be exposed to its

action for a suitable time: in fact, if the time is too long, too many heavy elements are formed, if it is too short, too few elements are formed. But, by “cooking” so to say the material to the right point one succeeds in obtaining something which has a certain resemblance with the experimental data. This resemblance arrives at such a point as to give, in the case of elements with high atomic number, a distribution of isotopes resembling the real one. In the region of light elements the result is, instead, contrary to the experimental one but one can think that a successive heat treatment, even at not an exceedingly high temperature, might have modified the situation.

As we have said, Gamow was not content with these results and has taken a further step, a very risky and almost certainly wrong step. Almost certainly wrong since the step one takes when, to explain the facts, one assumes very precise hypotheses. In that case, as is obvious, the more precise the hypotheses are, the more easily one demonstrates that they are wrong. At any way, Gamow resolved to determine the time in which the formation of elements described above has happened by resorting to the theory of the expansion of the Universe. This theory is connected with the theory of general relativity and we attempt to give a short account of it.

Unfortunately also for general relativity, as for other physical theories, there does not exist a single theory and this entails a certain freedom of choice, but at present this choice cannot be made on the reliable basis of experimental results. But if we base ourselves on the simplest one of the theories of relativity, that one without a cosmological term, we can construct as has been done, a theory of the expansion of the Universe according to the following general lines.

One starts from the hypothesis, which has at least the merit of being very simple, that the energy density (matter and radiation) is uniform in the entire Universe, at least when one averages over very large regions of it.

Furthermore one assumes that the space has a constant curvature; this means that the Universe is homogeneous not only with respect to the energy density but also with respect to its geometrical properties. From this hypothesis one can infer that the Universe at a certainly well-defined time has the shape of a sphere or that of a pseudo-sphere; for particular reasons, connected with the present matter density, one must choose the pseudo-sphere, which is a sphere with an imaginary radius and obviously it is not possible to represent it by a figure. But if for the moment we leave out of consideration the fact that the object of which we want to speak is a pseudo-sphere and not a sphere and furthermore if we limit ourselves to represent only three of its four dimensions (the time and two spatial coordinates) it will be possible (Fig. 6) to give an idea of how the Universe evolves expanding itself. In Fig. 6 the Universe is represented by circles whose radius is increasing with time.

If now, by using the formulas of general relativity, one makes calculations, one can find a relation which connects the velocity of expansion of the radius, $r = iu$,

of the pseudo-sphere with the energy density w :

$$\left(\frac{du}{dt}\right)^2 = c^2 + \frac{Kc^2}{3}u^2w. \quad (2)$$

This formula says that the square of the time derivative of the modulus of the radius u of the pseudo-sphere equals the square of the light velocity plus a term which contains the radius itself, the energy density w and a constant K related to the gravitational constant G through

$$K = 8\pi\frac{G}{c^4}$$

and having the value of about $2 \times 10^{-48}dy n^{-1}$. If we now want to use this formula to describe the expansion of the Universe when its radius is very small, we can see that the first term of the right hand side becomes negligible since the energy density increases much faster than the squared radius decreases. Then formula (2) can be simplified in the following way:

$$\left(\frac{du}{dt}\right)^2 = \frac{Kc^2}{3}u^2w. \quad (3)$$

At this point Gamow made an interesting remark: if one admits that, when formula (3) holds, the energy is essentially radiation energy, one can, using the formula, obtain a relation between temperature and time which contains only universal constants. In this way, one arrives at eliminating the arbitrariness due to the value of temperature at which nuclear reactions had taken place and given origin to the elements. This arbitrariness might have allowed one to obtain, to a certain extent, any result whatsoever. We give here, without proof, the formula which links temperature to the time elapsed since the instant when the Universe had infinitesimal size

$$T = \left(\frac{3}{4Kc^2\sigma}\right)^{1/4} \frac{1}{\sqrt{t}} \text{ degrees}. \quad (4)$$

In this formula, σ represents the Stefan's law constant, T the absolute temperature and t the time. As we have already pointed out, this is an approximate formula and holds for values of t not too large, for instance not exceeding a few million years.

If we substitute the constants appearing in formula (4) by their values, we obtain

$$T = \frac{1.52 \cdot 10^{10}}{\sqrt{t}} \text{ degrees} \quad (5)$$

from which one sees that when t is, for instance, equal to one second, the temperature is, as one could have imagined, enormously high: of the order of 10^{10} degrees. But it decreases very fast so that, after one thousand seconds, it is already reduced to the order of magnitude of one hundred million degrees and this value is low enough to stop strong nuclear phenomena.

Then the temperature varies with time following a well-defined law and the pseudo-radius of the Universe varies with an equally determined law (it is proportional to the squared time) and therefore only one parameter is left undetermined: the density of neutrons; Gamow intends, by working on this single arbitrary parameter, to succeed in predicting the distribution of the abundances of the elements and indeed in a certain sense he succeeds. As a matter of fact, he succeeds as long as one is content with a very rough analysis of his results, but as one tries to enter into details, immediately one runs into trouble and probably troubles would increase if it was possible to carry out this analysis which is extremely complicated to do.

The first difficulties are met already in the lower region of the periodic system as soon as one asks oneself a little in detail in what manner the elements are gradually forming themselves. As we have already said, the first nucleus to be formed will be that of hydrogen, then through the merging of a proton with a neutron deuterium will be formed and then with the addition of another neutron tritium, which will decay through a beta decay into helium three. By addition of a new neutron helium three changes into helium four. Already here one meets a little difficulty because helium three capturing neutrons tends to break rather than to form helium four, nevertheless one can still think that at least a small fraction of helium three changes by capture of a neutron into helium four. But at this point the difficulty one meets is much more important because the nucleus of mass five does not exist: if one would try to form it by addition of a neutron to helium four it would break to pieces creating an insurmountable barrier which prevents the successive formation of elements through the addition of neutrons.

In reality one can find some way to jump or even better to avoid this barrier, and it is the following: according to the formulas written above, in the time in which these phenomena should take place, the temperature, though already much decreased, is still on the order of 10^8 - 10^9 degrees and at such high temperatures nuclear reactions can still take place in a conspicuous way and are produced by the collisions among the nuclei which move under the effect of thermal agitation. Thus it is not impossible to think that a nucleus of mass six can be formed, without the preliminary existence of a nucleus of mass five, by making a nucleus of deuterium react directly with a nucleus of helium four. As a matter of fact, this reaction is extremely unlikely, but not impossible, therefore it is not excluded that a small amount of lithium six is formed allowing, through successive additions of neutrons the formation of heavier nuclei. And many other difficulties of this type are met; for instance, also the nucleus eight does not exist in any stable form and this missing step will be jumped by a device of the kind already described.

But the difficulties do not end here, another one which cannot be passed over in silence is that, if one assumes an initial density of neutrons large enough so that they can form heavy elements in nonnegligible quantities, one finds that the ratio between helium and hydrogen has nothing to do with the actual one: one will have more abundant helium than hydrogen contrary to experimental evidence.

So it only remains sadly to conclude that this theory is unable to explain the way in which the elements have been formed in time, and this after all is what one should have expected.

However, we must recognize the courage with which Gamow has set about constructing an attempt at a theory based on extremely determined hypotheses: the theory has failed and this means that some of his hypotheses are wrong, but the result he has obtained in this way (to be at least certain of having made a mistake) is certainly more remarkable than one that which could have been obtained from a theory so indefinite as to be able to explain a lot of experimental facts, exactly because of the great deal of arbitrariness contained in it, but that would not have made evident what are its incorrect points allowing them to be corrected and to proceed to the construction of new and more satisfactory theories.

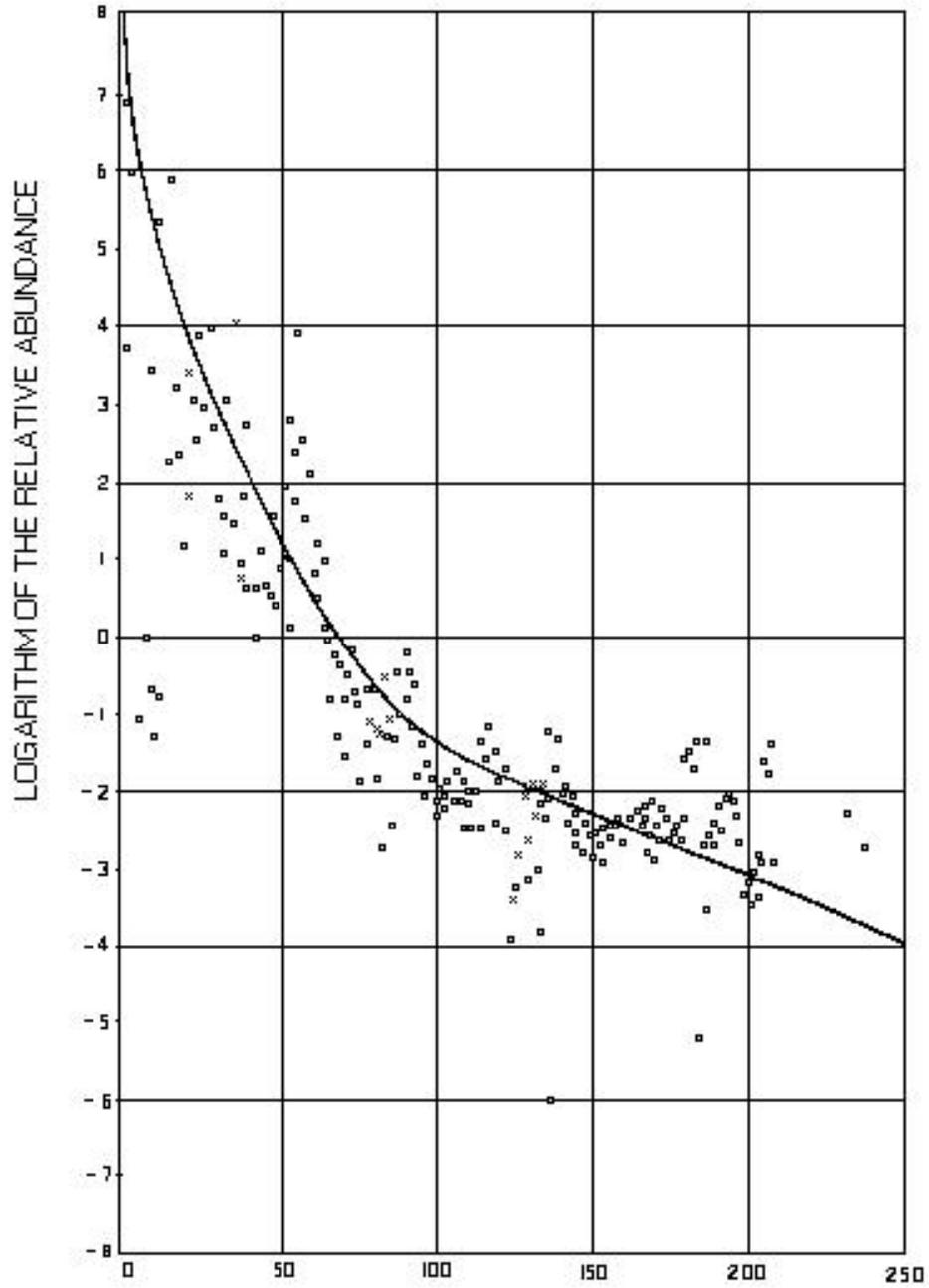


Figure 1. ATOMIC WEIGHT [This CAPTION really should be in the graphic as an axis label.]

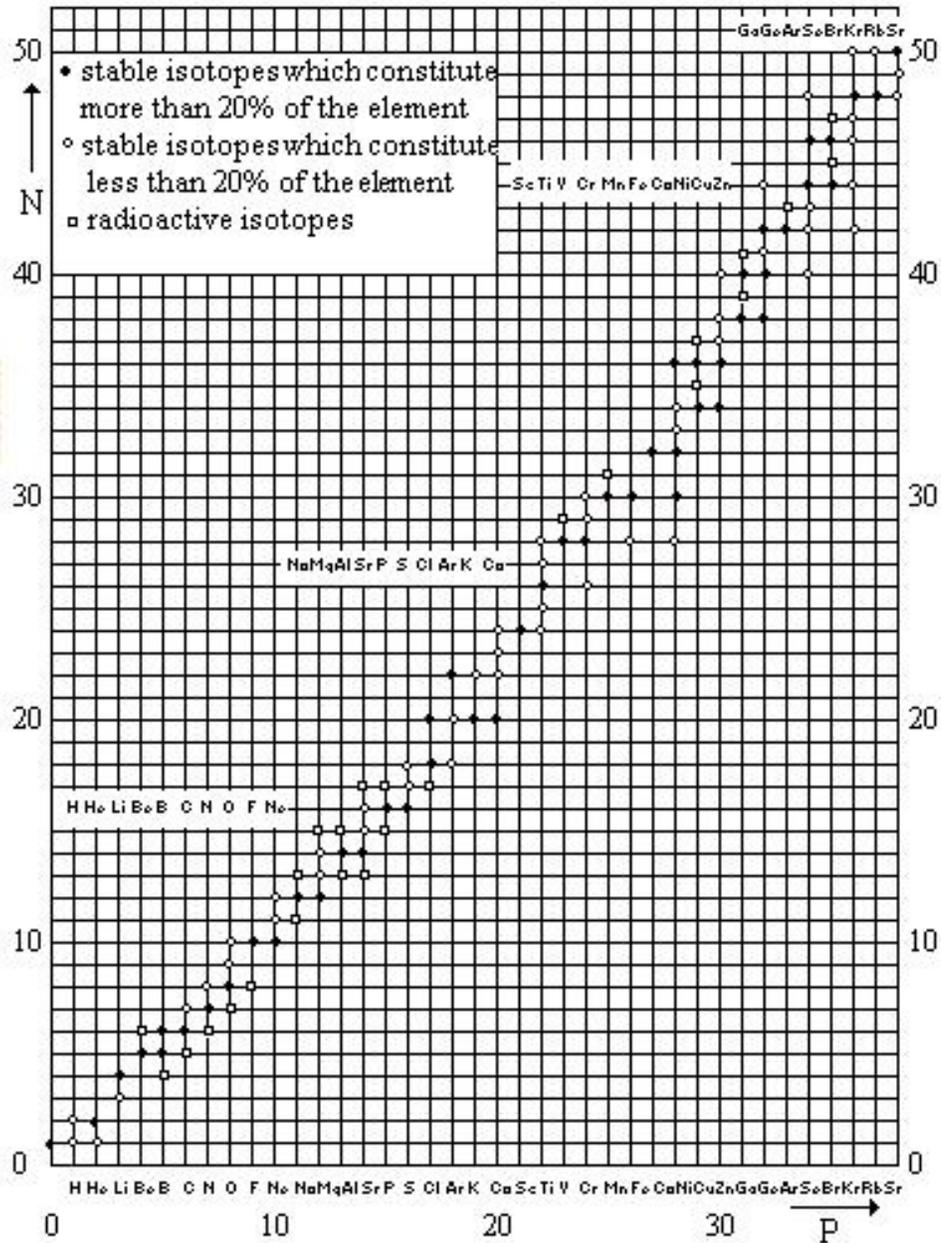


Figure 2.

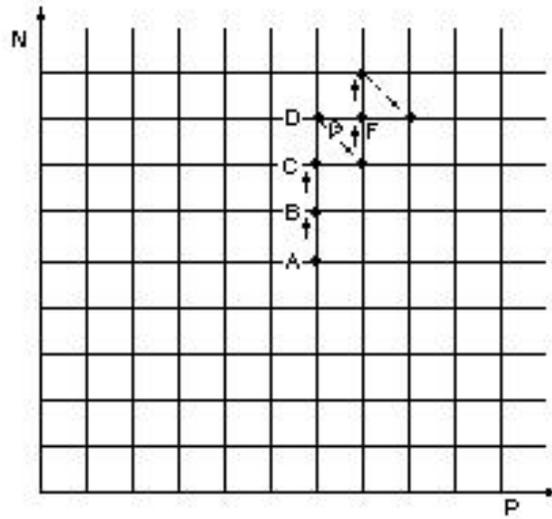


Figure 3.

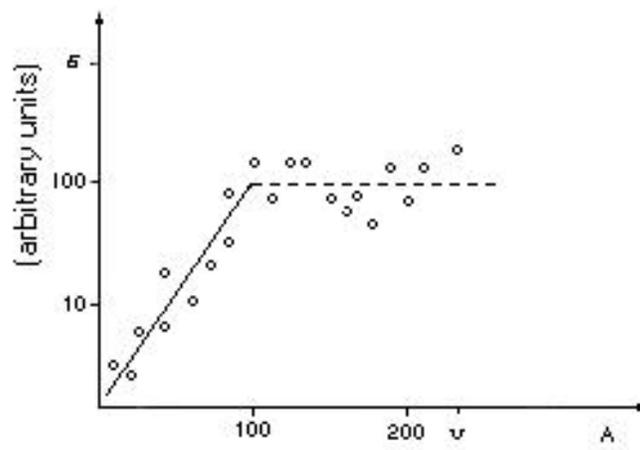


Figure 4.

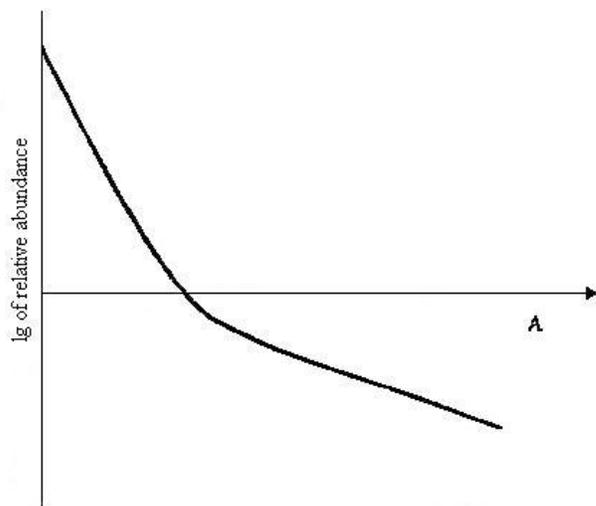


Figure 5.

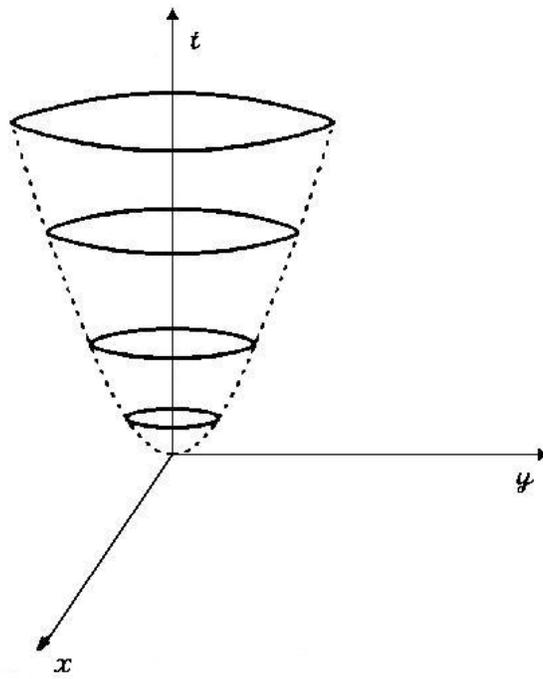


Figure 6.