ellipse evolute plot with normal lines

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The full discussion of this problem is at this website:
http://www34.homepage.villanova.edu/robert.jantzen/maple/ellipse/

We plot the ellipse with semi-axes plotted with its evolute and a family of equally spaced (in the parameter space) normal lines.

```
> restart : BasisFormat(false) : with(plots) : with(LinearAlgebra) :
```

The parametrized ellipse is: $x = a\cos(t), y = b\sin(t), t = 0..2\pi$:

Set the shape of the ellipse:

```
> a := 1 : b := 3/2 :
```

The parametrized curve:

```
> R(t) := <a cos(t), b sin(t)> : R(t)
```

$$\begin{bmatrix} \cos(t) \\ \frac{3\sin(t)}{2} \end{bmatrix} \tag{1}$$

The evolute is calulated elsewhere:

```
> Evolute(t) := (a^2 - b^2) <cos(t)^3/a, -sin(t)^3/b> : Evolute(t)
```

$$\begin{bmatrix} -\frac{5\cos(t)^3}{4} \\ \frac{5\sin(t)^3}{6} \end{bmatrix} \tag{2}$$

The normal to the ellipse is:

```
> NV(t) := <-2 cos(t)/sqrt(3 cos(t)^2 + 1), -sin(t)/sqrt(3 cos(t)^2 + 1)>
```

$$NV := t \mapsto \left\langle -\frac{2\cdot\cos(t)}{\sqrt{3\cdot\cos(t)^2+1}}, -\frac{\sin(t)}{\sqrt{3\cdot\cos(t)^2+1}} \right\rangle \tag{3}$$

The normal line is:

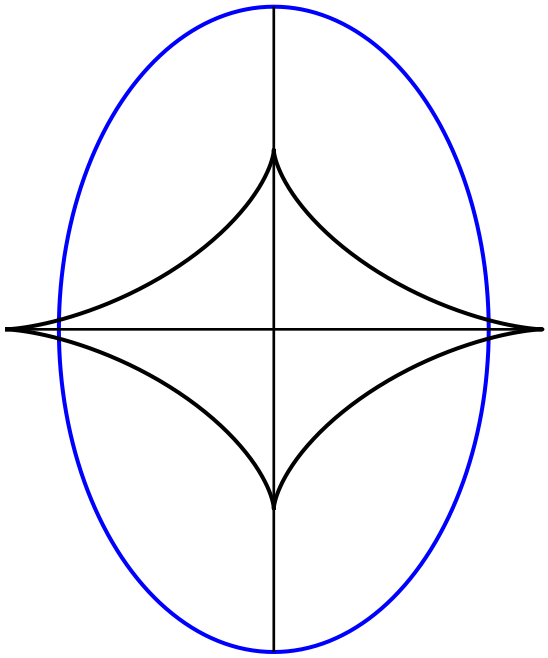
```
> NLINE := unapply(<cos(t) - 3 s cos(t)/sqrt(5 cos(t)^2 + 4), 3/2 sin(t) - 2 s sin(t)/sqrt(5 cos(t)^2 + 4)>, (t, s)) : NLINE(t, s)
```

$$\begin{bmatrix} \cos(t) - \frac{3\,s\cos(t)}{\sqrt{5\cos(t)^2+4}} \\ \frac{3\sin(t)}{2} - \frac{2\,s\sin(t)}{\sqrt{5\cos(t)^2+4}} \end{bmatrix} \tag{4}$$

We plot the curve and its evolute first.

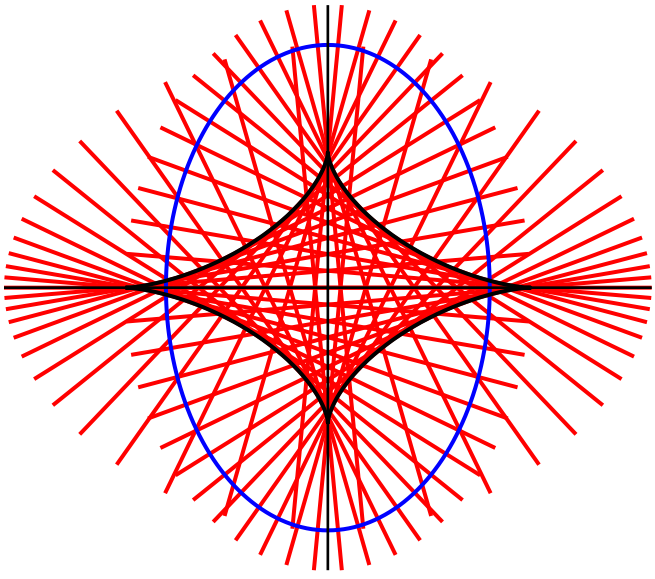
```
> Curveplot := plot([R(t)[1], R(t)[2], t = 0..2π], numpoints = 193, scaling = constrained, color = blue) :
  Eplot := plot([Evolute(t)[1], Evolute(t)[2], t = 0..2π], numpoints = 193, scaling = constrained, color
```

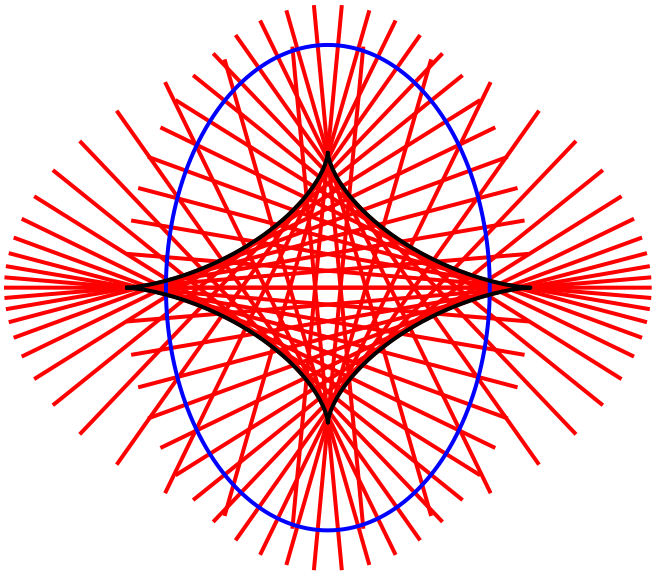
```
    = black) :
display(Curveplot, Eplot, tickmarks = [0, 0]);
```



We build up the list of normal line segments and display with the ellipse and its evolute.

```
> plotlist := NULL :
  frames := 50 :
  for i from 1 to frames do
   tfinal := i·2 π/frames :
   Nplot := plot([NLINE(tfinal, s)[1], NLINE(tfinal, s)[2], s = -0.25 ..3], numpoints = 193, scaling
       = constrained, color = red) :
   plotlist := plotlist, Nplot :
   od:
  display([plotlist, Curveplot, Eplot], scaling = constrained, insequence = false, axes = normal, tickmarks
       = [0, 0]);
  display([plotlist, Curveplot, Eplot], scaling = constrained, insequence = false, axes = normal, tickmarks
       = [0, 0], axes = none)
```





The second plot omits the axes.

Now for the black and white only version, with and without the axes.

```
> CurveplotBW := plot( [R(t)[1], R(t)[2], t = 0 ..2π], numpoints = 193, scaling = constrained, color
      = black) :
  Eplot := plot( [Evolute(t)[1], Evolute(t)[2], t = 0 ..2π], numpoints = 193, scaling = constrained, color
      = black) :
  display(CurveplotBW, Eplot, tickmarks = [0, 0]);
  plotlist := NULL :
  frames := 50 :
  for i from 1 to frames do
  tfinal := i·2 π/frames :
  Nplot := plot( [NLINE(tfinal, s)[1], NLINE(tfinal, s)[2], s = -0.25 ..3], numpoints = 193, scaling
      = constrained, color = black) :
  plotlist := plotlist, Nplot :
  od:
  display( [plotlist, CurveplotBW, Eplot], scaling = constrained, insequence = false, axes = normal, tickmarks
      = [0, 0]);
  display( [plotlist, CurveplotBW, Eplot], scaling = constrained, insequence = false, axes = normal, tickmarks
      = [0, 0], axes = none)
```