

(not for grade)

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC).

$$3x_1 + x_2 + x_3 + 6x_4 = 14$$

$$x_1 - 2x_2 + 5x_3 - 5x_4 = -7$$

$$4x_1 + x_2 + 2x_3 + 7x_4 = 17$$

- a) Write down the coefficient matrix A , the RHS matrix \vec{b} and the augmented matrix $C = \langle A \mid \vec{b} \rangle$ for this linear system of equations.
- b) With technology reduce this matrix C to its Reduced Row Echelon Form. Entered your matrix carefully.
- c) Write out the equations that correspond to the reduced matrix. Identify the leading variables (L) and the free variables (F) and solve. State your final solution in the vector form: $\langle x_1, x_2, x_3, x_4 \rangle = \dots$, etc.
- d) Check your solution with the Maple Tools, Tutors, Linear Algebra, Linear System Solving, Gauss-Jordan elimination popup.

► solution

$$a) A = \begin{bmatrix} 3 & 1 & 1 & 6 \\ 1 & -2 & 5 & -5 \\ 4 & 1 & 2 & 7 \end{bmatrix} \quad b = \begin{bmatrix} 14 \\ -7 \\ 17 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 1 & 1 & 6 & 14 \\ 1 & -2 & 5 & -5 & -7 \\ 4 & 1 & 2 & 7 & 17 \end{bmatrix}$$

$$b) C \xrightarrow[\text{Maple}]{\text{rref}} \begin{bmatrix} 1 & 0 & 1 & 1 & 3 \\ 0 & 1 & -2 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

L L F F
 $x_1 \ x_2 \ x_3 \ x_4$

$$c) \begin{aligned} x_1 + x_3 + x_4 &= 3 \rightarrow x_1 = 3 - x_3 - x_4 = 3 - t_1 - t_2 \\ x_2 - 2x_3 + 3x_4 &= 5 \rightarrow x_2 = 5 + 2x_3 - 3x_4 = 5 + 2t_1 - 3t_2 \\ 0 &= 0 \end{aligned}$$

$$L: x_1, x_2$$

$$F: x_3, x_4 \rightarrow x_3 = t_1, x_4 = t_2$$

$$\boxed{\langle x_1, x_2, x_3, x_4 \rangle = \langle 3 - t_1 - t_2, 5 + 2t_1 - 3t_2 \rangle}$$

$$d) \text{ Maple LinearSolveTutor: } \begin{aligned} t_1 &\rightarrow -t_3 \\ t_2 &\rightarrow -t_4 \end{aligned} \quad \text{agrees.}$$