MAT2500-01/03 21S Quiz 8 Print Name (Last, First) ______________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation when appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

Find the volume under the surface $z=x^2+y^2$ above the region of the plane between the 2 curves $y=\sqrt{3}\,x$, $x^2+y^2=4x$. First set up the three iterated integrals representing this volume and in each case let Maple evaluate the double integral:

a) in Cartesian coordinates integrating first in the vertical direction,
b) in Cartesian coordinates integrating first in the horizontal direction,
c) in polar coordinates.

In each case accompany your work with a new iteration diagram to justify your interation, a diagram shaded by equally spaced linear cross-sections and a typical one with bullet point endpoints labeled by the equation of the starting and stopping values of the integration variable for the inner integral and with an arrowhead midway indicating the variable's increasing direction.

d) Use Maple to evaluate each such integral exactly. Do they agree as they should?
e) What is the value of the volume numerically evaluated to 4 significant digits?
f) Use the Student[MultiCalculus]) command MultiInt to see which one of these three multistep evaluations is the least frightening. State your opinion. State the partial evaluation of the radial integration only for the polar coordinate version as a single integral over the polar angle.

solution

intersection: $3x^2=4x-x^2$, $4x^2-4x=0$, $x(x-1)=0$, $x=1,\ y=\sqrt{3}$!

a) $x^2+y^2=4x$

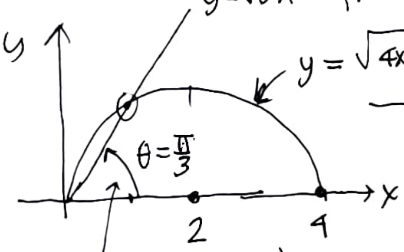
$x^2-4x+y^2=0$

$(x-2)^2-4+y^2=0$

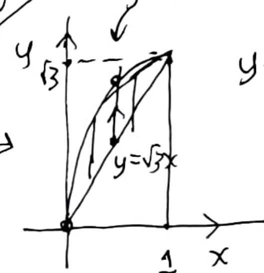
$(x-2)^2+y^2=4$

circle radius 2
center $(2,0)$

$y=\pm\sqrt{4x-x^2}$



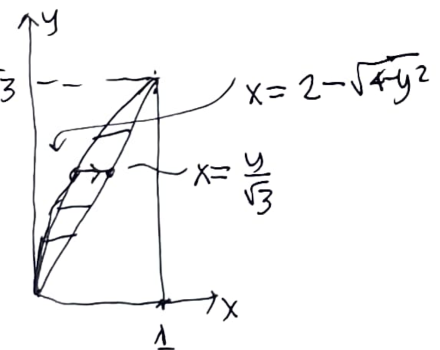
$\left(\theta=\arctan\sqrt{3}=\frac{\pi}{3}\right)$



$y=\sqrt{3}x\,..\,\sqrt{4x-x^2}$ while $x=0..1$

$$\boxed{V=\int_0^1\int_{\sqrt{3}x}^{\sqrt{4x-x^2}} x^2+y^2\,dy\,dx \overset{(d)}{=} 4\pi-7\sqrt{3} \overset{(e)}{\approx} 0.4420}$$

b) $x^2-4x+y^2=0 \to x=\frac{4\pm\sqrt{16-4y^2}}{2}=2\pm\sqrt{4-y^2}$



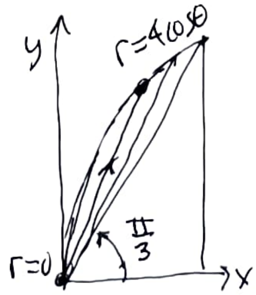
$x=2-\sqrt{4-y^2}\,..\,\frac{y}{\sqrt{3}}$ while $y=0..\sqrt{3}$

$$\boxed{\int_0^{\sqrt{3}}\int_{2-\sqrt{4-y^2}}^{y/\sqrt{3}} x^2+y^2\,dx\,dy \overset{(d)}{=} V}$$

c) $\cdot\, r^2=4(r\cos\theta) \to r^2-4r\cos\theta=0 \to r(r-4\cos\theta)=0$

$r=4\cos\theta$

$r=0\,..\,4\cos\theta$ while $\theta=\pi/3\,..\,\pi/2$



$$\boxed{V\overset{(d)}{=}\int_{\pi/3}^{\pi/2}\int_0^{4\cos\theta} \underbrace{(r^2)\,r}_{r^3}\,dr\,d\theta}$$

$$\underbrace{\frac{r^4}{4}\Big|_0^{r=4\cos\theta}}= 4^3\cos^4\theta$$

f) $\boxed{=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 64\cos^4\theta\,d\theta}$

You dont need MultiInt to see that the Cartesian outer integrals are of radical expressions composed with quadratics UGH! Polar coords win