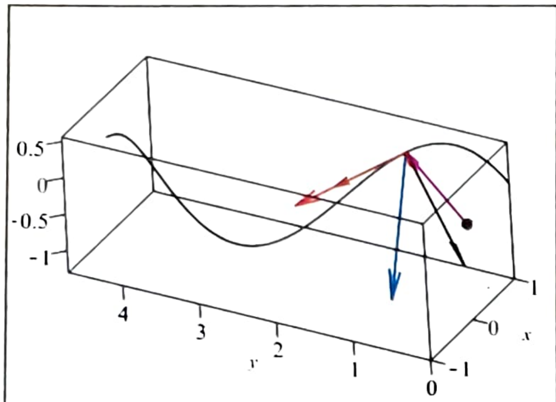
MAT2500-01/03 21S Quiz 4 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, IDENTIFYING expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.



Given the vector-valued function $\vec{r}(t) = \left\langle \cos(t), t, \frac{1}{2}\sin(2t) \right\rangle$ for the domain $t = 0 .. \frac{3\pi}{2}$ (no credit for unidentified expressions):

a) Evaluate $\vec{r}\,'(t)$, $\vec{r}\,''(t)$, $|\vec{r}\,'(t)|$, $\vec{T}(t)$ and remember to simplify your results.

b) Evaluate $\vec{r}\left(\frac{\pi}{3}\right)$, $\vec{r}\,'\left(\frac{\pi}{3}\right)$, $\vec{r}\,''\left(\frac{\pi}{3}\right)$, $\vec{T}\left(\frac{\pi}{3}\right)$ and remember to simplify your results.

c) Evaluate the exact angle θ in radians between $\vec{r}\,'\left(\frac{\pi}{3}\right)$ and $\vec{r}\,''\left(\frac{\pi}{3}\right)$ and a single decimal place approximation in degrees. Does it seem compatible with the figure, which shows the position vector from the origin, the first and second derivatives and the latter's projections with respect to the unit tangent? Why?

d) Evaluate the scalar projection $a_{||}$ of $\vec{a} = \vec{r}\,''\left(\frac{\pi}{3}\right)$ along $\vec{r}\,'\left(\frac{\pi}{3}\right)$.

e) Write the simplified equation for the plane containing the first and second derivatives as shown in the figure. What is its distance from the origin to 3 decimal places?

▶ **solution** too many to box:

a) $\vec{r} = \langle \cos t, t, \frac{1}{2}\sin 2t \rangle$

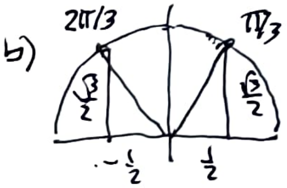
$\vec{r}\,' = \langle -\sin t, 1, \cos 2t \rangle$

$\vec{r}\,'' = \langle -\cos t, 0, -2\sin 2t \rangle$

$|\vec{r}\,'| = \sqrt{1 + \sin^2 t + \cos^2 2t}$

$\hat{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \frac{\langle -\sin t, 1, \cos 2t \rangle}{\sqrt{1 + \sin^2 t + \cos^2 2t}}$

b)



$\cos\frac{\pi}{3} = \frac{1}{2} = -\cos\frac{2\pi}{3}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} = \sin\frac{2\pi}{3}$

$\vec{r}(\frac{\pi}{3}) = \langle \frac{1}{2}, \frac{\pi}{3}, \frac{1}{2}(\frac{\sqrt{3}}{2}) \rangle = \langle \frac{1}{2}, \frac{\pi}{3}, \frac{\sqrt{3}}{4} \rangle$

$\vec{r}\,'(\frac{\pi}{3}) = \langle -\frac{\sqrt{3}}{2}, 1, -\frac{1}{2} \rangle = \frac{1}{2}\langle -\sqrt{3}, 2, -1 \rangle$

$|\vec{r}\,'(\frac{\pi}{3})| = \sqrt{1 + \frac{3}{4} + \frac{1}{4}} = \sqrt{2}$

$\hat{T}(\frac{\pi}{3}) = \frac{1}{2\sqrt{2}}\langle -\sqrt{3}, 2, -1 \rangle$

$\vec{r}\,''(\frac{\pi}{3}) = \langle -\frac{1}{2}, 0, -2(\frac{\sqrt{3}}{2}) \rangle = \langle -\frac{1}{2}, 0, -\sqrt{3} \rangle \equiv \vec{a}$

c) $|\vec{r}\,''(\frac{\pi}{3})| = \sqrt{\frac{1}{4} + 3} = \frac{\sqrt{13}}{2}$

$\hat{a} = \frac{2}{\sqrt{13}}\langle -\frac{1}{2}, 0, -\sqrt{3} \rangle = \frac{\langle -1, 0, -2\sqrt{3} \rangle}{\sqrt{13}}$

$\cos\theta = \hat{T}(\frac{\pi}{3}) \cdot \hat{a} = \frac{1}{2\sqrt{2}}\langle -\sqrt{3}, 2, -1 \rangle \cdot \frac{1}{\sqrt{13}}\langle -1, 0, -2\sqrt{3} \rangle$

$= \frac{1}{2\sqrt{2}\sqrt{13}}(\sqrt{3} + 2\sqrt{3}) = \frac{3\sqrt{3}}{2\sqrt{2}\sqrt{13}}$

$\boxed{\theta = \arccos\left(\frac{3}{2}\sqrt{\frac{3}{2 \cdot 13}}\right) \approx 59.4^\circ}$

60° is in the right ballpark, even not knowing what angle we are viewing the plane of $\vec{r}\,'$ and $\vec{r}\,''$.

d) $a_{||} = \hat{T}(\frac{\pi}{3}) \cdot \vec{a} = \frac{1}{2\sqrt{2}}\langle -\sqrt{3}, 2, -1 \rangle \cdot \frac{1}{2}\langle -1, 0, -2\sqrt{3} \rangle$

$= \frac{1}{4\sqrt{2}}(\sqrt{3} + 2\sqrt{3}) = \boxed{\frac{3}{4}\sqrt{\frac{3}{2}} = \frac{3}{8}\sqrt{6}}$

e) $\vec{n} = 4\vec{r}\,'(\frac{\pi}{3}) \times \vec{r}\,''(\frac{\pi}{3}) = \langle -\sqrt{3}, 2, -1 \rangle \times \langle -1, 0, -2\sqrt{3} \rangle$

$\underset{\text{Maple}}{=} \langle 4\sqrt{3}, 5, -2 \rangle$

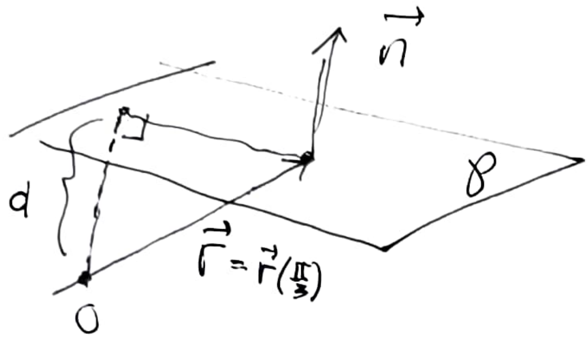
$0 = \vec{n} \cdot (\vec{r} - \vec{r}(\pi/3))$ combine

$= 4\sqrt{3}(x - \frac{1}{2}) + 5(y - \frac{\pi}{3}) - 2(z - \frac{\sqrt{3}}{4})$

$= \boxed{4\sqrt{3}x + 5y - 2z - (\frac{3\sqrt{3}}{2} + 5\pi/3) = 0}$ ugly!

Mat2500-001/003 2015 Quiz 4 answerkey (2)

e) continued. Plane : $\boxed{4\sqrt{3}x + 5y - 2z = \frac{3\sqrt{3}}{2} + \frac{5\pi}{3}}$



$d = |\hat{n}\cdot\vec{r}|$ project separation vector along normal direction

$\vec{n} = \langle 4\sqrt{3}, 5, -2\rangle$

$|\vec{n}| = \sqrt{3\cdot 16 + 25 + 4} = \sqrt{77}$, $\hat{n} = \dfrac{\langle 4\sqrt{3}, 5, -2\rangle}{\sqrt{77}}$

$\vec{r}(\frac{\pi}{3}) = \frac{1}{12}\langle 6, 4\pi, 3\sqrt{3}\rangle$

$\hat{n}\cdot\vec{r}(\frac{\pi}{3}) = \frac{1}{12\sqrt{77}} \underbrace{\langle 6, 4\pi, 3\sqrt{3}\rangle \cdot \langle 4\sqrt{3}, 5, -2\rangle}_{24\sqrt{3} + 20\pi - 6\sqrt{3}}$ combine $= 18\sqrt{3}$

$= \dfrac{\frac{3}{2}\sqrt{3} + \frac{5\pi}{3}}{\sqrt{77}} \approx 0{,}89277 \approx \boxed{0{,}893}$

I forgot this last part so was trying to squeeze the solution onto page 1. Oh well. Chalk it up to absent-minded bob.