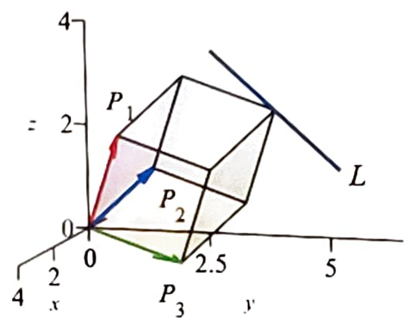
MAT2500-01/03 21S Quiz 3 Print Name (Last, First) ________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not subsitute for them.

Given three points $P_1(1,1,2)$, $P_2(-1,1,1)$, $P_3(3,3,0)$ and the parallelopiped formed from their three position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3$ as shown.



a) Find a normal vector $\vec{n}$ for the plane $\mathscr{P}$ through the origin which contains the first two points.

b) Write the simplified equation for this plane $\mathscr{P}$.

c) Write the parametrized equations of the line L through the vertex Q at the tip of the main diagonal of the parallelopiped which is orthogonal to this plane $\mathscr{P}$ as shown.

d) What are the coordinates of the point where the line L intersects this plane $\mathscr{P}$?

e) What is the area A of this plane face of the parallelopiped?

f) Let h be absolute value of the scalar projection of $\vec{r}_3$ along $\vec{n}$.

g) Evaluate the triple scalar product $\vec{r}_3 \cdot (\vec{r}_1 \times \vec{r}_2)$. Is this consistent with parts e), f)? Explain.

▶ solution

a) $\vec{r}_1 \times \vec{r}_2 = \langle 1,1,2\rangle \times \langle -1,1,1\rangle$

$\underset{\text{Maple}}{=} \boxed{\langle -1,-3,2\rangle = \vec{n}}$

b) $\vec{r}_0 = \langle 0,0,0\rangle$

$\boxed{0 = \vec{n}\cdot(\vec{r}-\vec{r}_0) = \langle -1,-3,2\rangle \cdot \langle x,y,z\rangle = -x-3y+2z}$

c) $\overrightarrow{OQ} = \vec{r}_1+\vec{r}_2+\vec{r}_3$ # $\vec{a} = \vec{n} = \langle -1,-3,2\rangle$

$\langle 1,1,2\rangle$
$+\langle -1,1,1\rangle$
$+\langle 3,3,0\rangle$
$= \langle 3,5,3\rangle = \vec{r}_4$

$\vec{r} = \vec{r}_4 + t\vec{n} = \langle 3,5,3\rangle + t\langle -1,-3,2\rangle$
$= \boxed{\langle 3-t, 5-3t, 3+2t\rangle = \langle x,y,z\rangle}$

d) substitute x,y,z in plane eqn by values along line:

$-(3-t)-3(5-3t)+2(3+2t) = 0$

$t+9t+4t-3-15+6 = 0$

$14t = 12 \rightarrow t = \frac{12}{14} = \frac{6}{7}$ backsub:

$(x,y,z) = (3-\frac{6}{7}, 5-\frac{3\cdot 6}{7}, 3+2(\frac{6}{7}))$
$= (\frac{21-6}{7}, \frac{35-18}{7}, \frac{21+12}{7}) = \boxed{(\frac{15}{7}, \frac{17}{7}, \frac{33}{7})}$

e) $A = |\vec{r}_1 \times \vec{r}_2| = |\vec{n}|$
$= \sqrt{1+9+4} = \boxed{\sqrt{14}}$

f) $\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{\langle -1,-3,2\rangle}{\sqrt{14}}$

$(\vec{r}_3)_{\parallel} = \hat{n}\cdot\vec{r}_3 = \frac{1}{\sqrt{14}}\langle -1,-3,2\rangle \cdot \langle 3,3,0\rangle$
$= \frac{1}{\sqrt{14}}(-1(3)-3(3)) = -\frac{12}{\sqrt{14}}$

$h = \left|\frac{-12}{\sqrt{14}}\right| = \boxed{\frac{12}{\sqrt{14}}} = \frac{12}{14}\sqrt{14} = \boxed{\frac{6}{7}\sqrt{14}}$

g) $\vec{r}_3 \cdot (\vec{r}_1 \times \vec{r}_2) = \langle 3,3,0\rangle \times \langle -1,-3,2\rangle$

$\underset{\text{Maple}}{=} \boxed{-12}$

The volume of the parallelopiped is

$V = A|(\vec{r}_3)_{\parallel}| = (\sqrt{14})\left|-\frac{12}{\sqrt{14}}\right| = 12$

This equals $|\vec{r}_3\cdot(\vec{r}_1\times\vec{r}_2)| = 12$

so they are consistent.