

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them, except for the cross product.

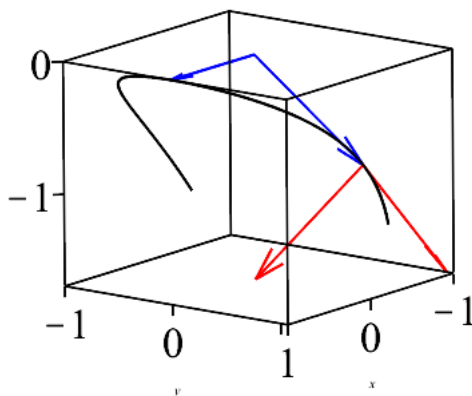
pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and hand this test sheet on top of your answer sheets as a cover page, with the first test page facing up:

"During this examination, all work has been my own. I have not accessed any of the class web pages or any other sites during the exam. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: _____

Date: _____



The parametrized curve segment $\vec{r}(t) = \left\langle \cos(t), \sin(t), 2 \ln\left(\cos\left(\frac{t}{2}\right)\right) \right\rangle$, $-2 \leq t \leq 2$.

is shown in the figure together with $\vec{r}(0)$ and $\vec{r}\left(\frac{\pi}{2}\right)$ and the first and second derivatives at the latter point on the curve.

a) Simplify $\vec{r}\left(\frac{\pi}{2}\right)$ using rules of logs for the last component.

b) Evaluate and simplify $\vec{v}(t) = \vec{r}'(t)$, $v(t) = |\vec{r}'(t)|$ and their values at $t = \frac{\pi}{2}$.

c) Your expression for $v(t)$ can be simplified to the square root of a perfect square by expressing all the terms which appear in terms of the cosine and sine and combining fractions. This allows you to find the exact arclength of the curve over the interval $0 \leq t \leq \frac{\pi}{2}$. Evaluate that exact value and its numerical approximation to 4 decimal places. Evaluate finally the arclength function starting at $t=0$ within this interval where all the trig functions which appear are nonnegative so we don't need to worry about absolute value signs.

d) Using the simplified result for $v(t)$ evaluate the unit tangent $\hat{T}(t)$ and $\hat{T}\left(\frac{\pi}{2}\right)$.

e) Evaluate and simplify $\vec{a}(t) = \vec{r}''(t)$, $a(t) = |\vec{r}''(t)|$ and their values at $t = \frac{\pi}{2}$.

f) Write the parametrized equations of the tangent line through $\vec{r}\left(\frac{\pi}{2}\right)$.

g) Evaluate $\vec{b}\left(\frac{\pi}{2}\right) = \vec{r}'\left(\frac{\pi}{2}\right) \times \vec{r}''\left(\frac{\pi}{2}\right)$. Then evaluate and simplify the unit binormal $\hat{B}\left(\frac{\pi}{2}\right) = \frac{\vec{b}\left(\frac{\pi}{2}\right)}{|\vec{b}\left(\frac{\pi}{2}\right)|}$ obtained by normalizing it.

h) Evaluate and simplify the unit normal $\hat{\mathbf{N}}\left(\frac{\pi}{2}\right) = \hat{\mathbf{B}}\left(\frac{\pi}{2}\right) \times \hat{\mathbf{T}}\left(\frac{\pi}{2}\right)$.

i) Write the simplified equation of the osculating plane through $\vec{\mathbf{r}}\left(\frac{\pi}{2}\right)$ containing the tangent vector and the second derivative there.

j) Evaluate the curvature $\kappa\left(\frac{\pi}{2}\right) = \frac{\left|\vec{\mathbf{r}}'\left(\frac{\pi}{2}\right) \times \vec{\mathbf{r}}''\left(\frac{\pi}{2}\right)\right|}{\left|\vec{\mathbf{r}}'\left(\frac{\pi}{2}\right)\right|^3}$ and its reciprocal, the radius of curvature $\rho\left(\frac{\pi}{2}\right)$.

k) Evaluate the scalar tangential projection $a_{\hat{\mathbf{T}}}\left(\frac{\pi}{2}\right)$ along $\hat{\mathbf{T}}\left(\frac{\pi}{2}\right)$ of the acceleration $\vec{\mathbf{a}}\left(\frac{\pi}{2}\right) = \vec{\mathbf{r}}''\left(\frac{\pi}{2}\right)$ and its scalar normal projection $a_{\hat{\mathbf{N}}}\left(\frac{\pi}{2}\right) = \hat{\mathbf{N}}\left(\frac{\pi}{2}\right) \cdot \vec{\mathbf{a}}\left(\frac{\pi}{2}\right)$ exactly. Does the sum of their squares equal the squared length of $\vec{\mathbf{a}}\left(\frac{\pi}{2}\right)$?

l) Evaluate the position vector of the center of the osculating circle: $\vec{\mathbf{C}}\left(\frac{\pi}{2}\right) = \vec{\mathbf{r}}\left(\frac{\pi}{2}\right) + \rho\left(\frac{\pi}{2}\right) \hat{\mathbf{N}}\left(\frac{\pi}{2}\right)$.

m) What is the angle between the first and second derivatives at $t = \frac{\pi}{2}$? Give the exact result and a one decimal place approximation in degrees.

Optional Teaser for Math Geeks.

This curve has reflection symmetry across the y -axis, namely $\langle x(-t), y(-t), z(-t) \rangle = \langle x(t), -y(t), z(t) \rangle$ but the expression for the signed arclength measured from $t=0$ is not obviously an odd function of t as it should be. Use trig and log identities to show that it is indeed an odd function.

► solution