

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation if appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

1. The temperature-humidity index I (or humidex, for short) is the perceived air temperature when the actual temperature is T and the relative humidity is h , so we can write $I = f(T, h)$. The following table of values of I is an excerpt from a table compiled by the National Oceanic & Atmospheric Administration. [Remember units in your responses.]

- (a) What is the value of $f(90, 50)$? What is its meaning?
- (b) For what value of h is $f(90, h) = 90$? Formulate this question in words.
- (c) Evaluate the average rate of change for $f(90, h)$ for the intervals $h = 40 \dots 50$ and then $h = 50 \dots 60$ and then average these to get a decimal value for the "instantaneous" rate of change of $f(90, h)$ at $h = 50$:

$$\frac{d}{dh} f(90, h) \Big|_{h=50}$$

- (d) Using this result, by how much would you expect the perceived temperature of 96° F to increase if the humidity increases from 50% to 52% at an actual temperature of 90° F?

Apparent temperature as a function of temperature and humidity

		Relative humidity (%)					
Actual temperature (°F)		20	30	40	50	60	70
T \ h	80	77	78	79	81	82	83
	85	82	84	86	88	90	93
	90	87	90	93	96	100	106
	95	93	96	101	107	114	124
	100	99	104	110	120	132	144

► solution

a) $f(90, 50) = 96^\circ$. When the actual temperature is 90° and the relative humidity is 50%, the perceived temperature is 96° .

b) $f(90, h) = 90 \rightarrow \boxed{h = 30}$ When the temperature is 90° , for what value of the humidity is the perceived temperature 90° ?

c)

		$\Delta h = 10$		
	T \ h	40	50	60
$\Delta T = 5$	<	85	88	
	<	90	93	96
		95		107

not requested bonus:

$\Delta I = 8 \quad \Delta I / \Delta T = 8/5 \rightarrow \left(\frac{\Delta I}{\Delta T}\right)_{avg} = \frac{1}{2} \left(\frac{8+11}{5}\right) = \frac{19}{10} = 1.9 \frac{^\circ F}{^\circ F}$

$\Delta I = 11 \quad \Delta I / \Delta T = 11/5 \rightarrow \left(\frac{\Delta I}{\Delta T}\right)_{avg} = \frac{1}{2} \left(\frac{8+11}{5}\right) = \frac{19}{10} = 1.9 \frac{^\circ F}{^\circ F}$

$\left(\frac{\Delta I}{\Delta T}\right)_{avg} = \frac{1}{2} \left(\frac{8+11}{5}\right) = \frac{19}{10} = 1.9 \frac{^\circ F}{^\circ F}$
(= $\frac{\partial I}{\partial T}(90, 50)$)

$\Delta I = 3 \quad \Delta I = 4$

$\frac{\Delta I}{\Delta h} = \frac{3}{10} \quad \frac{\Delta I}{\Delta h} = \frac{4}{10} \quad \left(\frac{\Delta I}{\Delta h}\right)_{avg} = \frac{1}{2} \left(\frac{3+4}{10}\right) = \boxed{0.35 \frac{^\circ F}{\%pt}} \quad \left(= \frac{\partial I}{\partial h}(90, 50) \right)$

d) For each $\%pt$ increase in humidity the perceived temperature increases by about 0.35° , so if the humidity increases by 2% , the temperature will increase about $\boxed{0.7^\circ}$ (from 96° to 96.7°).