MAT2500-01/02 20S Quiz 4 Print Name (Last, First) ______________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them.

Given the vector-valued function $\vec{r}(t) = \langle 4\sqrt{t}, t, t^2 \rangle$ for the domain $t \geq 0$ (no credit for unidentified expressions):

a) Evaluate $\vec{r}\,'(t)$, $\vec{r}\,''(t)$, $|\vec{r}\,'(t)|$, $\vec{T}(t)$ and remember to simplify your results.

b) Evaluate $\vec{r}(1)$, $\vec{r}\,'(1)$, $\vec{r}\,''(1)$, $\vec{T}(1)$ and remember to simplify your results.

c) Evaluate the exact angle θ in radians between $\vec{r}(1)$ and $\vec{r}\,'(1)$ and a single decimal place approximation in degrees. Does it seem compatible with the figure shown on the reverse side of this sheet? Why?

d) Evaluate the vector $\vec{w}$ which is the vector projection of $\vec{r}(1)$ orthogonal (perpendicular!) to $\vec{r}\,'(1)$.

e) **Optional.** Use the diagram on the reverse side to include a diagram of this projection.

▶ solution

a) $\vec{r} = \langle 4t^{1/2}, t, t^2 \rangle$

$\vec{r}\,' = \langle 2t^{-1/2}, 1, 2t \rangle$, $|\vec{r}\,'| = \underbrace{\sqrt{4/t + 1 + 4t^2} = \sqrt{\frac{4+t+4t^3}{t}}}_{\text{either acceptable}}$

$\vec{r}\,'' = \langle -t^{-3/2}, 0, 2 \rangle$

$\hat{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \frac{\langle 2t^{-1/2}, 1, 2t \rangle}{\sqrt{4/t + 1 + 4t^2}}$

b) $\boxed{\begin{array}{l} \vec{r}(1) = \langle 4, 1, 1 \rangle \\ \vec{r}\,'(1) = \langle 2, 1, 2 \rangle \\ \vec{r}\,''(1) = \langle -1, 0, 2 \rangle \\ \hat{T}(1) = \frac{1}{3}\langle 2, 1, 2 \rangle \end{array}}$

$\rightarrow |\vec{r}\,'(1)| = \sqrt{4+1+4} = 3$

c) $|\vec{r}(1)| = \sqrt{16+1+1} = \sqrt{18}$

$\widehat{r(1)} = \frac{\langle 4, 1, 1 \rangle}{\sqrt{18}}$ $\xrightarrow{\text{or}}$ $3\sqrt{2}$

$\cos\theta = \widehat{r(1)} \cdot \hat{T}(1) = \frac{\langle 4,1,1 \rangle}{\sqrt{18}} \cdot \frac{\langle 2,1,2 \rangle}{3}$

$= \frac{4(2) + 1(1) + 1(2)}{3 \cdot 3\sqrt{2}} = \frac{11}{9\sqrt{2}}$

$\theta = \arccos \frac{11}{9\sqrt{2}} \approx 30.204^\circ \approx \boxed{30.2^\circ}$

(Maple: $\frac{11\sqrt{2}}{18}$)

Although we dont know exactly how the particular 3d orientation of the plot distorts the angle, 30° looks very plausible in the present perspective!

d) $r(1)_{\parallel} = \hat{T}(1) \cdot \vec{r}(1) = \frac{1}{3}\langle 2,1,2 \rangle \cdot \langle 4,1,1 \rangle$

$= \frac{1}{3}(2(4) + 1(1) + 2(1)) = \frac{11}{3}$

$\overrightarrow{r(1)}_{\parallel} = \frac{11}{3}\hat{T}(1) = \frac{11}{3}\frac{1}{3}\langle 2,1,2 \rangle$

$= \frac{11}{9}\langle 2,1,2 \rangle$

aging eye saw 3 as 8!

$\overrightarrow{r(1)}_{\perp} = \vec{r}(1) - \overrightarrow{r(1)}_{\parallel}$

$= \langle 4, 1, 1 \rangle$

$- \langle \frac{22}{9}, \frac{11}{9}, \frac{22}{9} \rangle$

oops! careless

$= \langle \frac{36-22}{9}, \frac{9-11}{9}, \frac{9-22}{9} \rangle$

$\vec{w} = \boxed{\langle \frac{14}{9}, -\frac{2}{9}, -\frac{13}{9} \rangle = \frac{\langle 14, -2, -13 \rangle}{9}}$

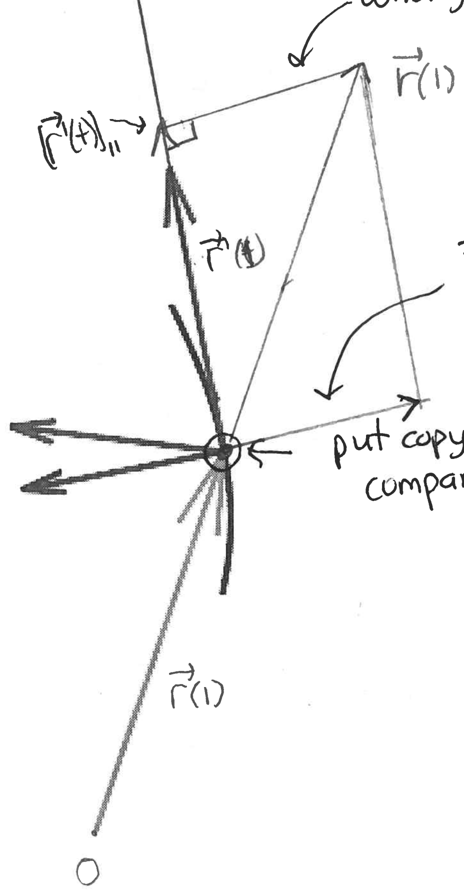
> $with(Student[VectorCalculus]) : BasisFormat(false) :$

> $r(t) := \langle 4\sqrt{t}, t, t^2 \rangle : r(t), r'(t), r''(t)$

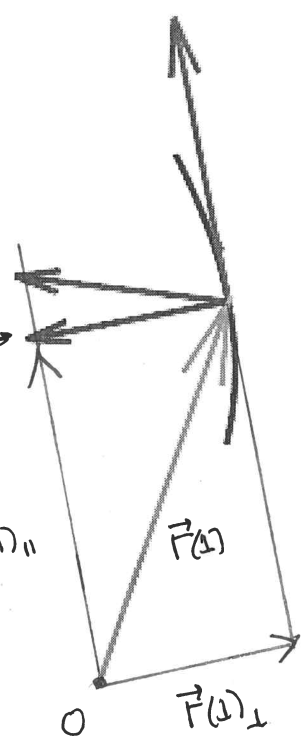
wild guess, but you can calculate:

$$\frac{|\vec{r}(1)_{\parallel}|}{|\vec{r}\,'(1)|} \approx 1.2$$

so I should have extended the projected point a bit farther up along the tangent line.



put copy of $\vec{r}(1)$ at tip of $\vec{r}(1)$ to compare with $\vec{r}\,'(1)$



or put line parallel to $\vec{r}\,'(1)$ at origin to project there

in fact this is aligned with $\vec{r}(1)_{\perp}$

we dont know distortion of angles from the view NOT orthogonal to the plane of $\vec{r}(1)$ and $\vec{r}\,'(1)$ but it cant be that great.