MAT2500-01/02 20S Test 4 Print Name (Last, First) ____________________________________|__

Show all work, including mental steps, in a clearly organized way that speaks for itself. Use proper mathematical notation, identifying expressions by their proper symbols (introducing them if necessary), and use EQUAL SIGNS and arrows when appropriate. Always SIMPLIFY expressions. BOX final short answers. LABEL parts of problem. Keep answers EXACT (but give decimal approximations for interpretation IF appropriate). Indicate where technology is used and what type (Maple, GC). Only use technology to CHECK hand calculations, not substitute for them. Explain in as many words as possible everything you are doing!

1. Given $\int_C (1+x^2 y)\,dx + x\,y\,dy$ over the closed curve which is the counterclockwise oriented semicircle $x^2 - 4x + y^2 = 0,\ y \ge 0$ completed by the diameter line segment along the x axis.

a) Evaluate the line integral directly using a polar coordinate parametrization of the semicircle. Be sure to state the simplified integrand before using technology to evaluate the iterated integral.

b) Use Green's Theorem to evaluate the equivalent double integral, again in polar coordinates. Be sure to state the simplified integrand before using technology to evaluate the iterated integral.

2. Given the two parametrized curves $r(t) = \langle t\cos(t), t\sin(t), t^2\rangle,\ t=0..2\pi$, and $r(t) = \langle t, 0, t^2\rangle,\ t=0..2\pi$, and the vector field

$F(x, y, z) = \langle 3x + y, x + 3y, 3z\rangle$.

a) Set up and simplify the line integrals of the vector field over each of these two paths between the same endpoints, then use technology to evaluate them.

b) Verify the differential condition that this vector field admit a scalar potential.

c) Solve the equations needed to construct a scalar potential.

d) Use the scalar potential to evaluate the line integral between these endpoints.

3. Consider the radial vector field $\vec{F}(x, y) = \dfrac{k\,\langle x, y\rangle}{x^2 + y^2}$ in the plane, where $k > 0$ is a constant. Show all the steps in evaluating and simplifying the partial derivatives.

a) Evaluate the magnitude of this vector field.

b) Evaluate and simplify $\hat{k} \cdot \text{curl}(\vec{F})$.

c) Evaluate and simplify $\text{div}(\vec{F})$.

d) Solve the equations which determine a potential function for this vector field in the plane.

e) Notice that this vector field is not defined at the origin where its magnitude has an infinite limit. Evaluate the (scalar) line integral of its outward normal component which appears in the Gauss version of Green's theorem

$$\int_C \vec{F} \cdot \hat{N}\,ds = \iint_R \text{div}(\vec{F})\,dA$$

for the region R inside a counterclockwise oriented circle $x^2 + y^2 = a^2$ to see the consequence of this single point for the field. How does this compare to the double integral side of the Gauss-Green theorem?

pledge

When you have completed the exam, please read and sign the dr bob integrity pledge and **and scan** this test sheet as a cover first page in the PDF scan of your lined paper hand work all on separate sheets.

"During this examination, all work has been my own. I give my word that I have not resorted to any ethically questionable means of improving my grade or anyone else's on this examination and that I have not discussed this exam with anyone other than my instructor, nor will I until after the exam period is terminated for all participants."

Signature: Date:

solution